

Physics 121.
Lecture 17.



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Physics 121.
Lecture 17.

- Course Information
- Topics to be discussed today:
 - Review of Rotational Motion
 - Rolling Motion
 - Angular Momentum
 - Conservation of Angular Momentum

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Physics 121.
Lecture 17.

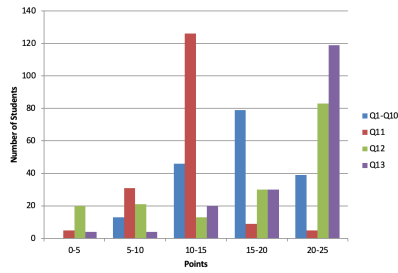
- Homework set # 6 is now available and is due on Saturday April 4 at 8.30 am.
- The grades for exam # 2 will be distributed via email on Monday March 30.
- You should pick up your exam in workshop this week. Please check it carefully for any errors.
- If you are not happy with the grading of your exam, you need to write down why you feel you deserve more points on a particular question, attach the note to your blue booklet(s), and return them to me before the end of lecture 20 (April 9).

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Results of individual questions on Exam # 2.

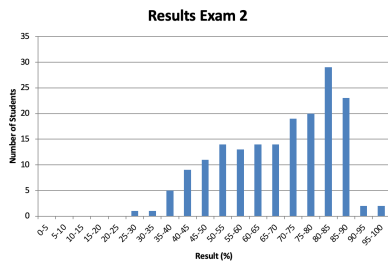


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Grade Distribution Exam # 2.

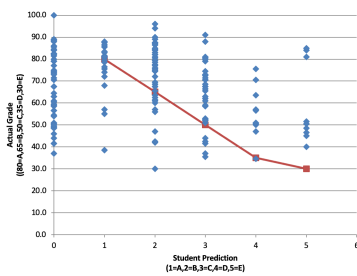


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Your prediction versus reality. Reality is better than your prediction.

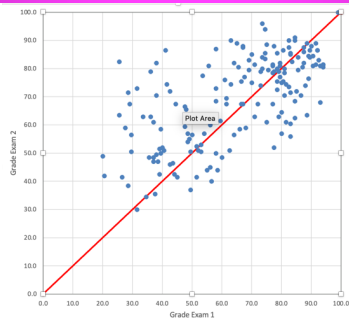


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Most of you did better on Exam # 2.

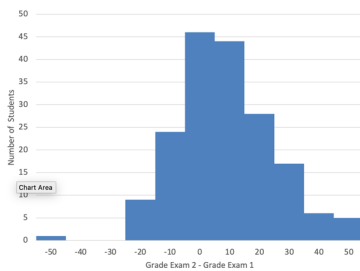


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Grade change between Exam # 2 and Exam # 1.



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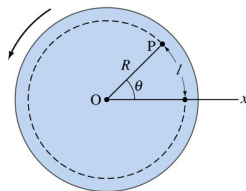
Rotational variables.
A quick review.

• The variables that are used to describe rotational motion are:

- Angular position θ
- Angular velocity $\omega = d\theta/dt$
- Angular acceleration $\alpha = d\omega/dt$

• The rotational variables are related to the linear variables:

- Linear position $l = R\theta$
- Linear velocity $v = R\omega$
- Linear acceleration $a = R\alpha$



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Rotational kinetic energy. A quick review.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} (\sum_i m_i r_i^2) \omega^2$$

- The kinetic energy is proportional to the square of the rotational velocity ω . Note: the equation is similar to the translational kinetic energy ($\frac{1}{2} m v^2$) except that instead of being proportional to the mass m of the object, the rotational kinetic energy is proportional to the **moment of inertia** I of the object:

$$I = \sum_i m_i r_i^2 \Rightarrow K = \frac{1}{2} I \omega^2$$

Note: units of I : kg m^2

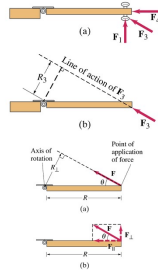
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Torque. A quick review.

- In general, the torque associated with a force F is equal to $|\vec{\tau}| = r F \sin \phi = |\vec{r} \times \vec{F}|$
- The arm of the force (also called the moment arm) is defined as $r \sin \phi$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle ϕ is 90° .



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Torque. A quick review.

- The torque τ of the force F is related to the angular acceleration α :

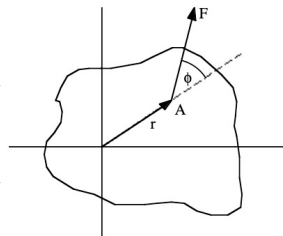
$$\vec{\tau} = I \vec{\alpha}$$

- This equation looks similar to Newton's second law for linear motion:

$$\vec{F} = m \vec{a}$$

- Corresponding variables for linear and rotational motion:

linear	rotational
mass m	moment I
force $\vec{F}$	torque $\vec{\tau}$



The direction of the torque defines the direction of the angular acceleration.

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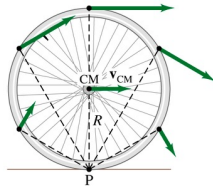
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Rolling motion. A quick review.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
 - Translational kinetic energy:

$$\frac{1}{2} M v_{CM}^2$$
 - Rotational kinetic energy:

$$\frac{1}{2} I_{CM} \omega^2$$
- Assuming the wheel does not slip:



$$\omega = \frac{v}{R}$$

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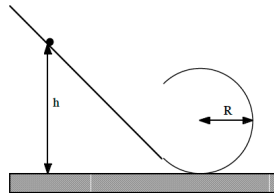
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How different is a world with rotational motion? Sample problem.

- Consider the loop-to-loop. What height h is required to make it to the top of the loop?
- First consider the case without rotation:
 - Initial mechanical energy: mgh .
 - Minimum velocity at the top of the loop is determined by requiring that $mv^2/R > mg$ or $v^2 > gR$.
 - The mechanical energy at the top of the loop is thus equal to

$$\frac{1}{2} mv^2 + 2mgR > \frac{5}{2} mgR$$
- Conservation of energy requires

$$h > \frac{5}{2} R$$



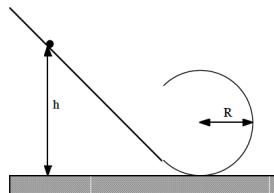
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How different is a world with rotational motion? Sample problem.

- What changes when the object rotates?
 - The minimum velocity at the top of the loop will not change.
 - The minimum translational kinetic energy at the top of the loop will not change.
 - But in addition to translational kinetic energy, there is now also rotational kinetic energy.
 - The minimum mechanical energy is at the top of the loop has thus increased.
 - The required minimum height must thus have increased.
- OK, let's now calculate by how much the minimum height has increased.



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How different is a world with rotational motion? Sample problem.

- The total kinetic energy at the top of the loop is equal to

$$K_f = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2}\left(\frac{I}{r^2} + M\right)v^2$$

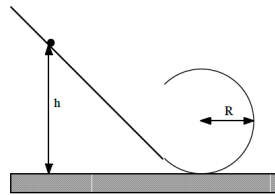
- This expression can be rewritten as

$$K_f = \frac{1}{2}\left(\frac{2}{5}M + M\right)v^2 = \frac{7}{10}Mv^2$$

- This is the minimum mechanical energy required to reach this point, and we can determine the minimum height:

$$h \geq \frac{27}{10}R$$

Note: without rotation $h \geq 25/10 R$!!!



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3 Minute 52 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 3 minute 52 second intermission.

- You can:

- Stretch out.
- Talk to your neighbors.
- Ask me a quick question.
- Enjoy the fantastic music.



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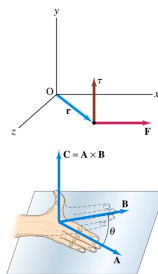
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Torque.

- The torque associated with a force is a vector. It has a magnitude and a direction.

- The direction of the torque can be found by using the right-hand rule to evaluate $\vec{r} \times \vec{F}$.

- For extended objects, the total torque is equal to the vector sum of the torque associated with each "component" of this object.



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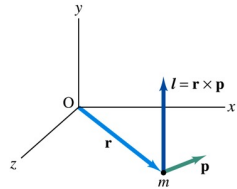
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Angular momentum.

- We have seen many similarities between the way in which we describe linear and rotational motion.
- Our treatment of these types of motion are similar if we recognize the following equivalence:

linear	rotational
mass m	moment I
force $\vec{F}$	torque $\vec{\tau} = \vec{r} \times \vec{F}$



- What is the equivalent to linear momentum?

Answer: **angular momentum.**

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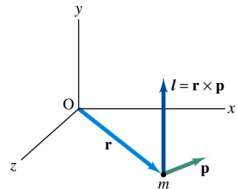
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Angular momentum.

- The **angular momentum** is defined as the vector product between the position vector and the linear momentum.

Note:

- Compare this definition with the definition of the torque.
- Angular momentum is a vector.
- The unit of angular momentum is $\text{kg m}^2/\text{s}$.
- The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
- Under certain circumstances the angular momentum of a system is conserved!



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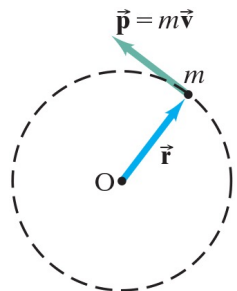
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Angular momentum.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius $\vec{r}$ and the linear momentum $\vec{p}$:

$$L = mvr = mr^2 \frac{v}{r} = I\omega$$

- Note: compare this with the linear momentum $p = mv$!



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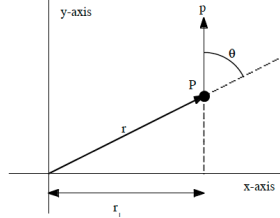
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Angular momentum.

- An object does not need to carry out rotational motion to have an angular momentum.
- Consider a particle P carrying out linear motion in the xy plane.
- The angular momentum of P (with respect to the origin) is equal to

$$\vec{L} = \vec{r} \times \vec{p} = mrv \sin \theta \hat{k} = pr_{\perp} \hat{k}$$

and will be constant (if the linear momentum is constant).



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Conservation of angular momentum.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = m \left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v} \right) = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times m\vec{a} = \vec{r} \times \sum \vec{F} = \sum \vec{\tau}$$

- Consider what happens when the net torque is equal to 0:

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow L = \text{constant}$$

- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.
- Note: notice again the similarities between linear and rotational motion.

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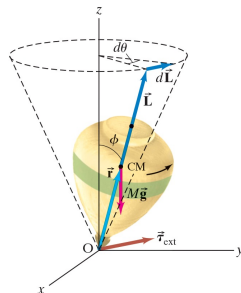
Conservation of angular momentum.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$$

is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).

- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).



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Conservation of angular momentum.

- Ignoring the mass of the bicycle wheel, the external torque will be close to zero if we use the center of the disk as our reference point.
- Since the external torque is zero, angular momentum thus should be conserved.
- I can change the orientation of the wheel by applying internal forces. In which direction will I need to spin to conserve angular momentum?



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Done for today!
More about rotational motion on Thursday.



The Orion Nebula from CFHT

Credit & Copyright: [Canada-France-Hawaii Telescope](#), [J.-C. Cuillandre \(CFHT\)](#), [Cepium](#)

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