

Physics 121.
Lecture 13.



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Physics 121.
Lecture 13.

- Course Information
- Topics to be discussed today:
 - Conservation of linear momentum (a brief review)
 - One- and two-dimensional collisions (elastic and inelastic)

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Physics 121.
Course Information.

- Homework set # 5 is now available on the WEB and will be due next week on Saturday morning, March 21, at 8.30 am.
- To download the collision videos:
 - OSX: use control-click while pointing to the movie links to download the linked file.
 - Windows: use right-click while pointing to the movie links to download the linked file.
- The most effective way to work on the assignment is to tackle 1 - 2 problems a day.

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Midterm Exam # 2. March 24, 2026.

- Midterm Exam # 2 is scheduled for Tuesday March 24 between 8 am and 9.20 pm.
- The material covered on Exam # 2 will Chapters 6, 7, 8, and 9.
- The format will be the same as the format of Exam # 1.
- I will review the material covered on the exam during my lecture on March 19.

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The center of mass (a review).

- The position of the center of mass of a system of particles along the x-axis is defined as

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{1}{M} \sum_i m_i x_i$$

and similar expressions for the y and z positions.

- The motion of the center of mass is determined by the external forces acting on the system:

$$M \vec{a}_{cm} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net, ext}$$

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Linear momentum (a review).

- The product of the mass and velocity of an object is called the **linear momentum** $\vec{p}$ of that object.
- In the case of an extended object, we find the total linear momentum by adding the linear momenta of all of its components:

$$\vec{P}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i = M \vec{v}_{cm}$$

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Linear momentum (a review).

- The change in the linear momentum of the system can now be calculated:

$$\frac{d\vec{p}_{cm}}{dt} = M \frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i = \vec{F}_{net,ext}$$

- This relation shows us that if there are no external forces, the total linear momentum of the system will be constant (independent of time).

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Now we have all the tools to really understand lift!

Sorry Bernoulli!

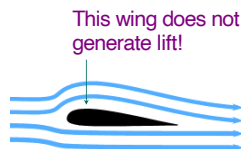
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Generating lift. Who needs Bernoulli?

- Most people who claim to understand the generation of lift, will rely on the Bernoulli effect to explain the principle of flight.
- If you are one of them, I am sorry to have to tell you that you do not understand the principle of flight.
- Although the Bernoulli effect has been used for many decades to explain the principle of flight, the Bernoulli effect can only account for a few percent of the lift generated by a wing, and it can not account for phenomena such as inverted flight.



<http://www.ua.washington.edu/faculty/leberhardt/lift.htm>

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NY Times, January 2004. 100 Year Anniversary of Flight.

What keeps them up there?

- "it is disconcerting that physicists and aeronautical engineers still passionately debate the fundamental issues"
- Jef Raskin, one of the creators of the Macintosh, was sent to the principal's office when he argued with his science teacher that the Bernoulli effect could not explain why paper airplanes fly. According to the science teacher paper airplanes use a different scientific principle to fly.



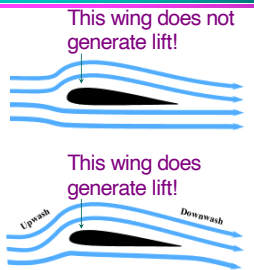
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Generating lift. Who needs Bernoulli?

- In order for a wing to generate lift, (producing an upward force) it must force the air down.
- The vertical component of the momentum associated with the downwash, must be balanced by the vertical component associated with the lift generated by the wing.
- Thus, unless the air is forced downwards, the wing will not generate lift.



<http://www.aa.washington.edu/faculty/leberhardt/lift.htm>

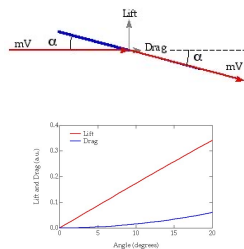
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Generating lift. Who needs Bernoulli?

- The lift of a wing is generated by the downwards deflection of air.
- Consider an elastic collision between an air molecule of mass m and velocity V . The change in the vertical component of the momentum of the wing as a result of this single collision is equal to:
$$p_{lift} = mV \sin(\alpha)$$
- The lift generated will increase if V increases, if the air density increases, and/or if the angle of attack increases.



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Generating lift. An example.

- Consider a Piper Warrior. This plane has a maximum weight of 2450 lb, and the required lift on takeoff is about 11,000 N.
- On takeoff, the airspeed is 60 knots = 31 m/s. Assume that on takeoff the angle of attack is 5°.
- To generate this lift requires that

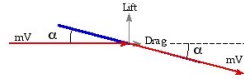
$$\text{Lift} = \frac{dp_{\text{lift}}}{dt} = \frac{dm}{dt} V \sin(\alpha)$$

- This requires that

$$\frac{dm}{dt} = \frac{\text{Lift}}{V \sin(\alpha)} = 4100 \frac{\text{kg}}{\text{s}}$$



www.newpiper.com



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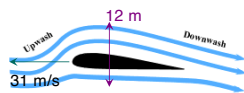
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Generating lift. Certainly not a surface phenomenon.

- The Warrior has a wing span of 35.0 ft (10.7 m) and a wing surface area of 170 ft² (15.8 m²).
- The density of air at sea level is about 1 kg/m³.
- In order to generate an air deflection rate of 4100 kg/s, the height of the layer of air involved must be about 12 m.
- Clearly, the generation of lift is not a surface phenomenon.



The Rochester Flying Club



<http://www.aas.washington.edu/faculty/leberhardt/lift.htm>

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Generating lift. What does it take to lift a Boeing 737?

- At 6.02 am on 3/22/03, CO 282 took off from San Antonio (TX). For this particular flight, the weight of the plane was 106,000 pounds and the rotation speed was 131 knots (numbers provided by Mark d'Arpino, the first officer on this flight).
- For this flight:
 - $dm/dt = 80,000 \text{ kg/s}$ (80,000 m³/s)
 - Note: $(dm/dt)/M = 1.7$ for this 737 compared to 3.7 for the Warrior.
 - The wingspan of this 737 is 94.9 ft, which indicates that a 41 m thick layer of air is involved in the generation of lift.



Photographer:
B. Leibowitz



Photographer:
M. Abbott

Pictures from airliners.net

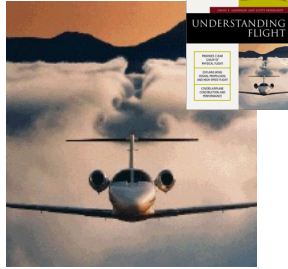
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Generating lift.

- The effect of the downwash of the wing of this citation, flying low over a fog layer, is clearly visible.
- More details about the generation of lift can be found in “*Understanding Flight*” by Anderson and Eberhardt. Note: Anderson is a retired high-energy physicist.



Photograph by Paul Bowen, courtesy of Cessna Aircraft, Co.

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Back to more mundane topics: collisions.

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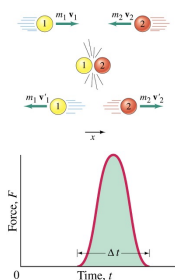
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Collisions. The collision force.

- During a collision, a strong force is exerted on the colliding objects for a short period of time.
- The collision force is usually much stronger than any external force.
- The result of the collision force is a change in the linear momentum of the colliding objects.
- The change in the momentum of one of the objects is equal to

$$\vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt$$



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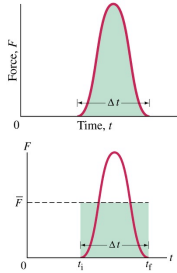
Collisions. The collision impulse.

- If we measure the change in the linear momentum of an object, we will obtain information about the integral of the force acting on it:

$$\vec{p}_f - \vec{p}_i = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt$$

- The integral of the force is called the collision impulse $\vec{J}$:

$$\vec{J} = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt$$



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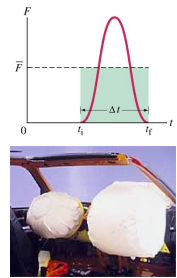
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Collisions. The collision impulse.

- Consider you are involved in a collision: you first move with 55 mph and after the collision you are at rest.
- The change in momentum is thus fixed and the collision impulse is also fixed:

$$\vec{J} = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) dt$$

- What happens to you depends on the magnitude of the force! An increase in time dt results in a reduction of the force.



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Collisions. The collision impulse.

- Increasing the time required to come to a stop reduced the average force.
- This reduction in the average force can mean the difference between life and death.
- The human body can tolerate accelerations up to 10 - 15 times the gravitational acceleration.
- An acceleration of 10g brings an object traveling at 55 mph to rest over a distance of 3 m (9 feet).



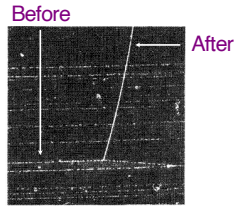
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Collisions. The collision impulse.

- Interactions between sub-atomic particles are usually studied by comparing their momenta before and after an interaction.
- The change in their momenta provides us with information about the collision impulse.
- Determining the force from the collision impulse required a knowledge of the time dependence of the interaction.



Collision between protons.

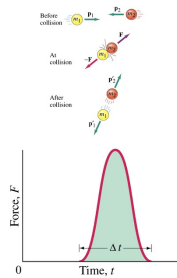
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Elastic and inelastic collisions.

- If we consider both colliding object, then the collision force becomes an internal force, and the total linear momentum of the system must be conserved if there are no external forces acting on the system.
- Collisions are usually divided into two groups:
 - Elastic collisions:** kinetic energy is conserved.
 - Inelastic collisions:** kinetic energy is NOT conserved.



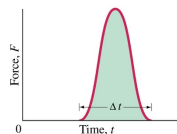
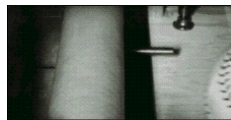
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Elastic and inelastic collisions.

- Kinetic energy does not need to be conserved during the time period that the collision force is acting on the system. The kinetic energy may become 0 J for a short period of time.
- During the time period that the collision force is non-zero, some or all of the initial kinetic energy may be converted into potential energy (for example, the potential energy associated with deformation).



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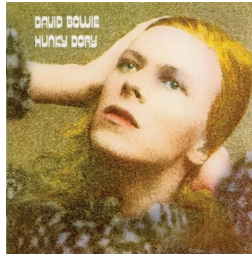
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4 Minute 14 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 14 second intermission.

- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.



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Elastic collisions in one dimension.

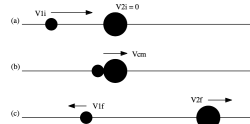
- Consider the elastic collision show in the Figure.
- Conservation of linear momentum requires that

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

- Conservation of kinetic energy requires that

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Two equations with two unknown!



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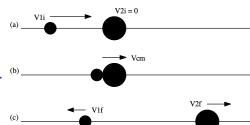
Elastic collisions in one dimension.

- The solution for the final velocity of mass m_1 is:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

- The solution for the final velocity of mass m_2 is:

$$v_{2f} = \frac{m_1}{m_2} (v_{1i} - v_{1f}) = \frac{2m_1}{m_1 + m_2} v_{1i}$$



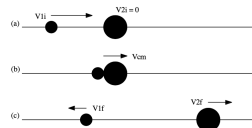
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Elastic collisions in one dimension. Special cases.

- $m_1 = m_2$:
 - $v_{1f} = 0 \text{ m/s}$
 - $v_{2f} = v_{1i}$
- $m_2 \gg m_1$:
 - $v_{1f} = -v_{1i}$
 - $v_{2f} = (2m_1/m_2) v_{1i}$
- $m_1 \gg m_2$:
 - $v_{1f} = v_{1i}$
 - $v_{2f} = 2v_{1i}$



Note: the motion of the center of mass is not changed due to the collision.

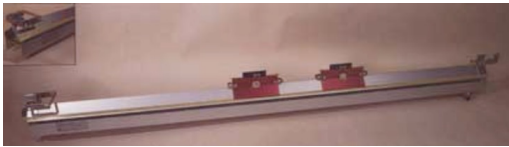
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Elastic collisions in one dimension.

- We can use an air track to study elastic collisions.
- The velocity of the carts can be determined if we measure the length of time required to pass through the photo gates: velocity = length/time.



TEL-Atomic, <http://www.telatomic.com/at.html>

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Elastic collisions in one dimension.

- Let us focus on specific examples where one cart (cart 2) is at rest:
 - $M_1 = M_2$: $v_{1f} = 0 \text{ m/s}$, $v_{2f} = v_{1i}$
 - $M_1 = 2M_2$: $v_{1f} = (1/3)v_{1i}$, $v_{2f} = (4/3)v_{1i}$
 - $2M_1 = M_2$: $v_{1f} = -(1/3)v_{1i}$, $v_{2f} = (2/3)v_{1i}$



TEL-Atomic, <http://www.telatomic.com/at.html>

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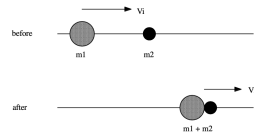
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Inelastic collisions in one dimension.

- In inelastic collisions, kinetic energy is not conserved.
- A special type of inelastic collisions are the completely inelastic collisions, where the two objects stick together after the collision.
- Conservation of linear momentum in a completely inelastic collision requires that

$$m_1 v_i = (m_1 + m_2) v_f$$



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Inelastic collisions in one dimension.

- The final velocity of the system is equal to

$$v_f = \frac{m_1}{m_1 + m_2} v_i$$

- The final kinetic energy of the system is equal to

$$\begin{aligned} K_f &= \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} (m_1 + m_2) \left(\frac{m_1}{m_1 + m_2} v_i \right)^2 \\ &= \frac{m_1}{m_1 + m_2} K_i \end{aligned}$$

- Note: not all of the kinetic energy can be lost, even in a completely inelastic collision, since the motion of the center of mass must still be present. Only if our reference frame is chosen such that the center-of-mass velocity is zero, will the final kinetic energy in a completely inelastic collision be zero.

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Inelastic collisions in one dimension.

- Let us focus on one specific example of a procedure to measure the velocity of a bullet:

- We shoot a 0.3 g bullet into a cart.
- The final velocity of the cart is measured, and conservation of linear momentum can be used to determine the velocity of the bullet:

$$v_i = \frac{(M_1 + M_2)}{M_1} v_f$$



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Conceptual Questions.

- Let's practice what we learned (and read).



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Done for today!
Thursday: collisions in 2D.



White Boat Rock on Mars

Credit: [Mars Exploration Rover Mission, JPL, NASA](#)

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