Physics 121.
Lecture 20.

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Physics 121.
Lecture 20.

- Course Information
- Topics to be discussed today:
  - Requirements for Equilibrium (a brief review)
  - Stress and Strain
  - Introduction to Harmonic Motion

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Physics 121.
Lecture 20.

- Homework set # 7 is due on Saturday morning, April 11, at 8.30 am. This assignment has two components:
  - WeBWorK (75%)
  - Video analysis (25%): you can calculate the angular momentum quickly by using the expression for the vector product in terms of the components of the individual vectors (the x and y components of the position and velocity of the puck).
- Homework set # 8 is now available. This assignment contains only WeBWorK questions and will be due on Saturday morning, April 18, at 8.30 am.
- Exam # 3 will take place on Tuesday April 28.

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And now something completely different!
Harmonic motion.

- We will continue our discussion of mechanics with the discussion of harmonic motion (simple and complex). This material is covered in Chapter 14 of our textbook.
- Chapter 14 will be the last Chapter included in the material covered on Exam # 3 (which will cover Chapters 10, 11, 12, and 14).
- Note: **We will not discuss the material discussed in Chapter 13 of the textbook, dealing with fluids, and this material will not be covered on our exams.**

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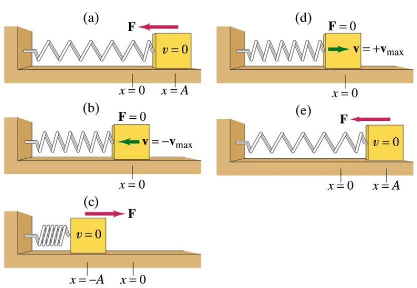
Harmonic motion.
Motion that repeats itself at regular intervals.



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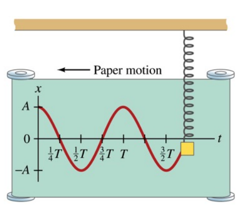
Simple harmonic motion.

Phase Constant

Amplitude

$x(t) = A\cos(\omega t + \phi)$

Angular Frequency



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Simple harmonic motion.

- Instead of the angular frequency ω the motion can also be described in terms of its period T or its frequency ν.
- The period T is the time required to complete one oscillation:

$$x(t) = x(t + T)$$

or

$$A\cos(\omega t + \phi) = A\cos(\omega t + \omega T + \phi)$$

- In order for this to be true, we must require $\omega T = 2\pi$. The period T is thus equal to $2\pi/\omega$.

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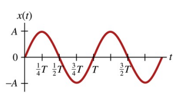
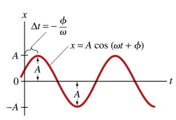
Simple harmonic motion.

- The frequency of the oscillation is the number of oscillations carried out per second:

$$\nu = 1/T$$

- The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 s^{-1}.





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Simple harmonic motion. What forces are required?

Consider we observe simple harmonic motion. The observation of the motion can be used to determine the nature of the force that generates this type of motion. In order to do this, we need to determine the acceleration of the object carrying out the harmonic motion:

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$$

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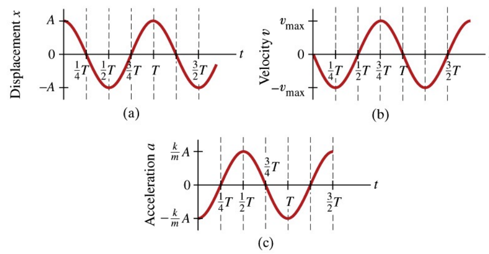
Simple harmonic motion. What forces are required?



Note: maxima in displacement correlate with minima in acceleration.

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Simple harmonic motion. What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m:

$$\omega = \sqrt{\frac{k}{m}}$$

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Simple harmonic motion. What forces are required?

- We conclude:

Simple harmonic motion is the motion executed by a particle of mass m, subject to a force F that is proportional to the displacement of the particle, but opposite in sign.

- Any force that satisfies this criterion **can** produce simple harmonic motion. If more than one force is present, you need to examine the net force, and make sure that the net force is proportional to the displacements, but opposite in sign.

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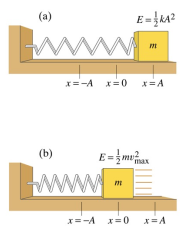
Simple harmonic motion.
Total energy.

- During the motion of a block on a spring, there is a continuous conversion of energy:
  - **The potential energy:**
    $$U(t) = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$
  - **The kinetic energy:**
    $$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$$
  - **The mechanical energy:**
    $$E(t) = U(t) + K(t) = \frac{1}{2}kA^2$$



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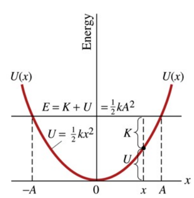
Simple harmonic motion.
Total energy.

- Since the mechanical energy is independent of time, we conclude that the mechanical energy of the system is constant!
- If we know the total mechanical energy of the system, we can determine the region of oscillation. This region is constraint by the fact that the kinetic energy is always positive, and the potential energy is thus constraint by the mechanical energy ($U \leq E$).



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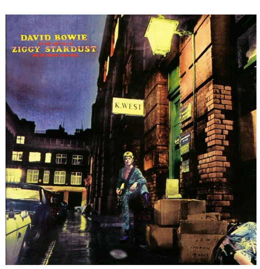
4 Minute 38 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 38 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



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Simple Harmonic Motion (SHM).
The torsion pendulum.

- What is the angular frequency of the SHM of a torsion pendulum:
  - When the base is rotated, it twists the wire and the wire generated a torque which is proportional to the the angular twist:

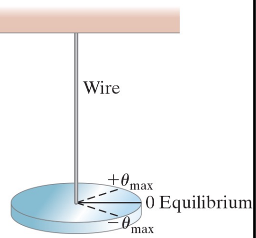
    $$\tau = -K\theta$$

    The torque generates an angular acceleration α:

    $$\alpha = \frac{d^2\theta}{dt^2} = \frac{\tau}{I} = -\frac{K}{I}\theta$$

    The resulting motion is harmonic motion with an angular frequency

    $$\omega = \sqrt{\frac{K}{I}}$$



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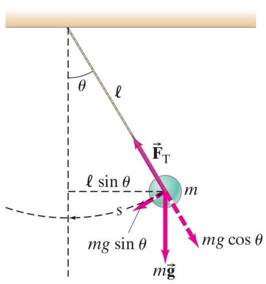
Simple Harmonic Motion (SHM).
The simple pendulum.

- Calculate the angular frequency of the SHM of a simple pendulum.
- A simple pendulum is a pendulum for which all the mass is located at a single point at the end of a massless string.
- There are two forces acting on the mass: the tension T and the gravitational force mg.
- The tension T cancels the radial component of the gravitational force.



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Simple Harmonic Motion (SHM).
The simple pendulum.

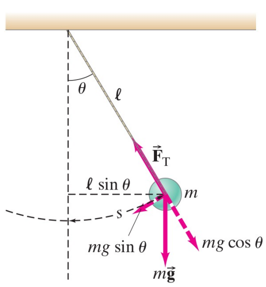
- The net force acting on the mass is directed perpendicular to the string and is equal to

  $$F = -mg\sin\theta$$

  The minus sign indicates that the force is directed opposite to the angular displacement.
- When the angle θ is small, we can approximate $\sin\theta$ by θ :

  $$F = -mg\theta = -mg\frac{x}{l}$$

- Note: the force is again proportional to the displacement.



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Simple Harmonic Motion (SHM).
The Simple Pendulum.

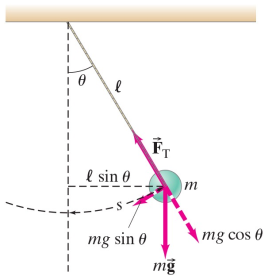
- The equation of motion for the pendulum is thus

$$F = m\frac{d^2x}{dt^2} = -\frac{mg}{l}x$$

or

$$\frac{d^2x}{dt^2} = -\frac{g}{l}x$$

- The equation of motion is the same as the equation of motion for a SHM, and the pendulum will thus carry out SHM with an angular frequency $\omega = \sqrt{g/l}$.



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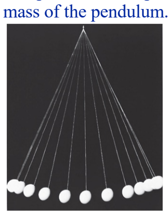
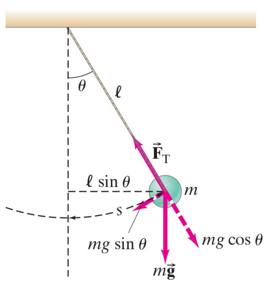
Simple Harmonic Motion (SHM).
The Simple Pendulum.

- The period of the pendulum is thus $T = 2\pi/\omega = 2\pi\sqrt{l/g}$.
- Note: the period is independent of the mass of the pendulum.





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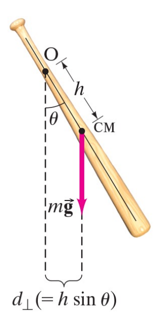
Simple Harmonic Motion (SHM).
The physical pendulum.

- In a realistic pendulum, not all mass is located at a single point.
- The motion carried out by this realistic pendulum around its rotation point O can be determined by determining the total torque with respect to this point:

$$\tau = -mgh\sin\theta$$

- If the angle θ is small, we can approximate the torque by

$$\tau = -mgh\theta$$



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Simple Harmonic Motion (SHM).
The physical pendulum.

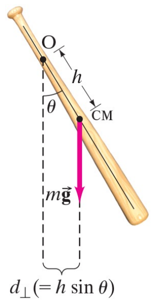
- The angular acceleration α is related to the torque:

$$\tau = I\alpha$$

- The equation of motion for the angular acceleration α is given by

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{\tau}{I} = -\frac{mgh}{I}\theta$$

- This again is an equation for SHM with an angular frequency $\omega = \sqrt{mgh/I}$.



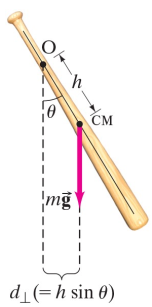
Simple Harmonic Motion (SHM).
The physical pendulum.

- The period of the physical pendulum is equal to

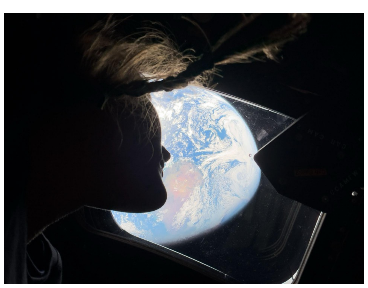
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgh}}$$

- We can double check our answer by requiring that the simple pendulum is a special case of the physical pendulum ($h = l$ and $I = ml^2$):

$$T = 2\pi\sqrt{\frac{ml^2}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$$



Done for today!
Next week: more harmonic motion!



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