Problem 1

The solution to this problem is similar to the solution to problem 4 on set 2, except that in addition to the pulling force T_3 we have to include the friction force. The first step is to calculate he acceleration of the system. Assume the acceleration is a then we know that the net force on the blocks must be

$$F_{net} = (m_1 + m_2 + m_3)a \tag{1}$$

The net force can also be expressed in terms of the pulling and friction forces:

$$F_{net} = T_3 - \mu_k(m_1 + m_2 + m_3)g \tag{2}$$

where $\mu_k m_i g$ is the friction force on mass m_i. By combining these two equations we can determine the acceleration a:

$$a = \frac{T_3 - \mu_k(m_1 + m_2 + m_3)g}{m_1 + m_2 + m_3} = \frac{T_3}{m_1 + m_2 + m_3} - \mu_k g \tag{3}$$

The net force on block m_1 is equal to

$$F_1 = m_1 a = \frac{m_1 T_3}{m_1 + m_2 + m_3} - \mu_k m_1 g \tag{4}$$

The net force on block m_1 can also be expressed in terms of the tension T_1 and the friction force acting on block 1:

$$F_1 = T_1 - \mu_k m_1 g \tag{5}$$

By combining these two expressions we see that

$$T_1 = \frac{m_1 T_3}{m_1 + m_2 + m_3} \tag{6}$$

We can follow the same procedure for block 2 where we find that the net force must be equal to

$$F_2 = m_2 a = \frac{m_2 T_3}{m_1 + m_2 + m_3} - \mu_k m_2 g \tag{7}$$

The second expression for the net force on block 2, expressed in terms of the tension in the strings and the friction force, is equal to

$$F_2 = T_2 - T_1 - \mu_k m_2 g \tag{8}$$

By combining these two expressions, and using the expression for T_1, we can obtain the following expression for T_2:

$$T_2 = F_2 + T_1 + \mu_k m_2 g = \frac{m_2 T_3}{m_1 + m_2 + m_3} - + \mu_k m_2 g + \frac{m_1 T_3}{m_1 + m_2 + m_3} + \mu_k m_2 g = \frac{(m_1 + m_2)T_3}{m_1 + m_2 + m_3} \tag{9}$$

Problem 2
There are three forces acting on each rider:

1. The gravitational force, which is directed in the negative vertical direction and has a magnitude of mg.
2. The friction force, which is directed in the positive vertical direction and must have a magnitude of mg if the riders do not slip down the wall.
3. The normal force, which is directed in the horizontal direction, directed towards the center of the cylinder, and with a magnitude of mv^2/r.

The friction force is proportional to the normal force:

$$F_{friction} = \mu_s N = \mu_s \frac{mv^2}{r} \tag{10}$$

Since the friction force must have a magnitude of mg we conclude that

$$\mu_s \frac{mv^2}{r} = mg \tag{11}$$

or

$$v = \sqrt{\frac{gr}{\mu_s}} \tag{12}$$

The period of the rider must be

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{gr}{\mu_s}}} = 2\pi\sqrt{\frac{\mu_s r}{g}} \tag{13}$$

The minimum number of revolutions per minute required to prevent the riders from sliding down is thus equal to

$$\text{revolutions/minute (rpm)} = \frac{60}{T} = \frac{60}{2\pi\sqrt{\frac{\mu_s r}{g}}} = \frac{30}{\pi\sqrt{\frac{g}{\mu_s r}}} \tag{14}$$

Problem 3

When we apply the external force F, the system will accelerate with an acceleration a where a is equal to

$$a = \frac{F}{m_! + m_2} \tag{15}$$

The net force on block 1 in the horizontal direction must thus be equal to

$$F_{net,1} = m_1 a = \frac{m_1}{m_! + m_2} F \tag{16}$$

In order for block 1 not to slide down, the friction force must have the same magnitude as the weight of block 1. Thus

$$\mu_s N = m_1 g \tag{17}$$

or

$$N = \frac{m_1 g}{\mu_s} \tag{18}$$

The normal force and the applied force are the only two forces that act on block 1 in the horizontal direction and we must thus require that

$$F_{net,1} = F - N = F - \frac{m_1 g}{\mu_s} = \frac{m_1}{m_! + m_2} F \tag{19}$$

or

$$F\left(1 - \frac{m_1}{m_! + m_2}\right) = \frac{m_2}{m_! + m_2} F = \frac{m_1 g}{\mu_s} \tag{20}$$

The minimum force F must therefore be equal to

$$F = \frac{m_1 + m_2}{m_2} \frac{m_1 g}{\mu_s} \tag{21}$$

Problem 4

The force exerted on my block is shown in Fig. 1.

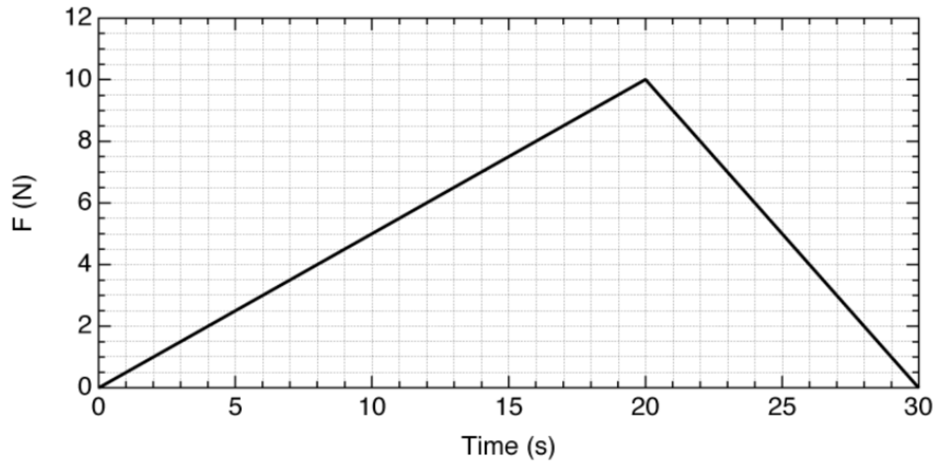

Figure 1: The force on my block as function of time.

The maximum force that can be applied to the block without it starting to move is equal to

$$F_{max} = \mu_s N = \mu_s mg = 0.089 \cdot 1.5 \cdot 9.81 = 1.31 \text{ N} \tag{22}$$

The time dependence of the force during the first 20 s is given by

$$F_{applied}(t) = \frac{10}{20}t = \frac{1}{2}t \tag{23}$$

The time at which the force reaches a magnitude of 1.31 N is 2.62 s.
Once the block is moving, the friction force will be kinetic friction with a magnitude equal to

$$f_{kinetic} = \mu_k N = \mu_k Mg = 0.048 \cdot 1.5 \cdot 9.81 = 0.71 \text{ N} \tag{24}$$

The net force on the block is thus equal to

$$F_{net}(t) = F_{applied}(t) - f_{kinetic} = \frac{1}{2}t - 0.71 \tag{25}$$

The acceleration of the block is thus equal to

$$a(t) = \frac{f_{net}}{M} = \frac{\frac{1}{2}t - 0.71}{1.5} = 0.33t - 0.47 \tag{26}$$

Since the block starts to move at $t = 2.62$ s, we will use this as our reference time. The velocity at this time is equal to 0 m/s. The change in the velocity between $t = 2.62$ s and $t = 20$ s is equal to

$$\Delta v = \int_{2.62}^{20} a(t)dt = \int_{2.62}^{20} (0.33t - 0.47)dt = (0.165t^2 - 0.47t)\Big|_{2.62}^{20} = 56.7 \text{ m/s} \tag{27}$$

Since the velocity initially is 0 m/s, the velocity at time t = 20 s is 56.7 m/s. Between t = 20 s and t = 30 s, the applied force is equal to

$$F_{applied} = 10 - \frac{10}{10}(t - 20) = 30 - t \tag{28}$$

The net force on the block during this time interval is thus equal to

$$F_{net} = F_{applied} - f_{kinetic} = 30 - t - 0.71 = 29.29 - t \tag{29}$$

The acceleration of the block is thus equal to

$$a = \frac{F_{net}}{M} = \frac{29.29 - t}{1.5} = 19.5 - 0.67t \tag{30}$$

The change in the velocity between t = 20 s and t = 30 s is equal to

$$\Delta v = \int_{20}^{30} a dt = \int_{20}^{30} (19.5 - 0.67t) dt = (19.5t - \frac{1}{2}0.67t^2)\Big|_{2.62}^{20} = 27.5 \text{ m/s} \tag{31}$$

Since the initial velocity is 56.7 m/s, the final velocity is 84.2 m/s.

Problem 5
Consider the system shown in Fig. 2.

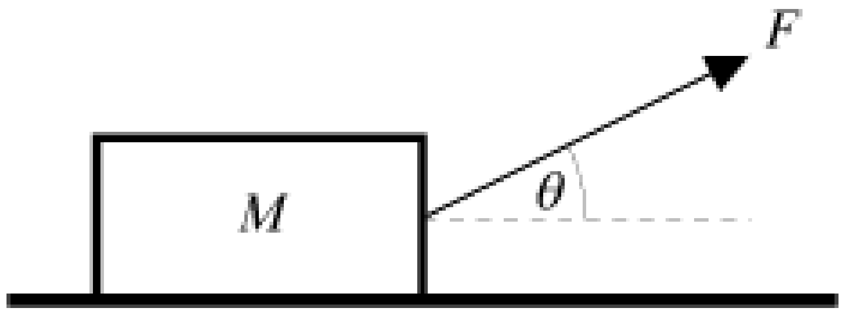

Figure 2: A force pulling block.

Since the block is not moving in the vertical direction, the net force in this direction must be equal to 0. The net force in this direction is equal to

$$F_y = N + F\sin\theta - Mg = 0 \tag{32}$$

where N is the normal force exerted by the floor on the block. The normal force is thus equal to

$$N = Mg - F\sin\theta \tag{33}$$

Using the normal force N, we can now determine the kinetic friction force between the block and the floor:

$$f_{kin} = \mu_k N = \mu_k(Mg - F\sin\theta) \tag{34}$$

The net force in the horizontal direction acting on the block is thus equal to

$$F_x = F\cos\theta - f_{kin} = F\cos\theta - \mu_k(Mg - F\sin\theta) = F(\cos\theta + \mu_k\sin\theta) - \mu_k Mg \tag{35}$$

The acceleration of the block is equal to

$$a = \frac{F_x}{M} = \frac{F}{M}(\cos\theta + \mu_k\sin\theta) - \mu_k g \tag{36}$$

The speed after t seconds, assuming the block start from rest, is equal to

$$v = at = \left(\frac{F}{M}(\cos\theta + \mu_k\sin\theta) - \mu_k g\right)t \tag{37}$$

Problem 6
When the system remains at rest, the net force on each block must be zero. Consider the force diagram shown in Fig. 3.

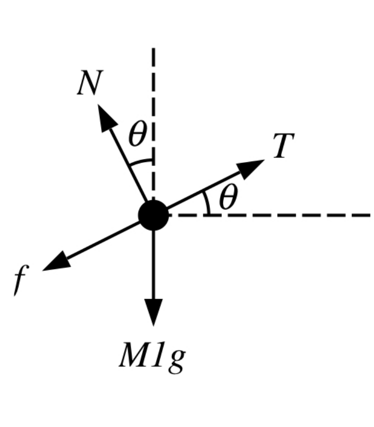

Figure 3: Block M_1 is at rest on the inclined plane.

Since the net force on block 2 must be zero, we immediately conclude that the tension T must be equal to $M_2 g$. To describe the force on block 1, we will use as our coordinate axes an axis parallel to the plane and an axis perpendicular to the plane. The net force in the direction perpendicular to the plane is

$$F_\perp = N - M_1 g \cos\theta = 0 \tag{38}$$

This requirement allows us to determine the normal force N:

$$N = M_1 g \cos\theta \tag{39}$$

The net force in the direction parallel to the plane is

$$F_\parallel = T - M_1 g \sin\theta - f_{static} = 0 \tag{40}$$

The required static friction force is thus equal to

$$f_{static} = T - M_1 g \sin\theta = M_2 g - M_1 g \sin\theta \tag{41}$$

Taking into consideration that the static friction force has a maximum value, the requirement of no motion can be written as:

$$f_{static} = M_2 g - M_1 g \sin\theta \leq \mu_s N = \mu_s M_1 g \cos\theta \tag{42}$$

or

$$\frac{M_2 g - M_1 g \sin\theta}{M_1 g \cos\theta} = \frac{M_2 - M_1 \sin\theta}{M_1 \cos\theta} \leq \mu_s \tag{43}$$

If block 1 is in motion, the friction force will be the kinetic friction force. This force will be directed in a direction opposite the direction of motion. The force acting on the blocks when block 1 is moving up the incline are shown in Fig. 4.

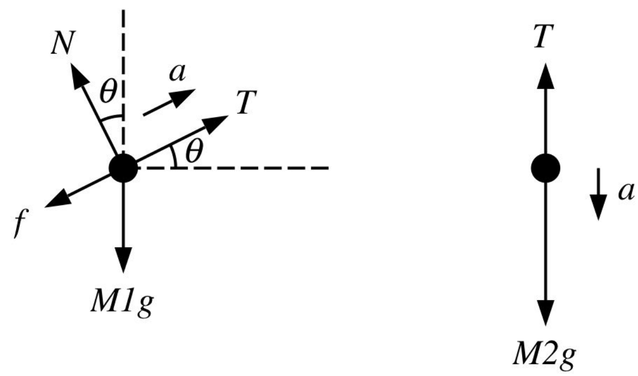

Figure 4: Block M_1 is moving up the inclined plane.

It is very important to make sure that the directions of the forces are used consistently. For example, if block 1 moves up the incline, a positive acceleration corresponds to block 2 moving down. The net force on block 2 is equal to

$$F_2 = M_2 g - T = M_2 a \tag{44}$$

The net force on block 1 is equal to

$$F_1 = T - M_1 g \sin\theta - f_k = T - M_1 g \sin\theta - \mu_k M_1 g \cos\theta = T - M_1 g(\sin\theta + \mu_k \cos\theta) = M_1 a \tag{45}$$

Adding both equations, we can eliminate T and determine a:

$$M_2 g - M_1 g(\sin\theta + \mu_k \cos\theta) = (M_1 + M_2)a \tag{46}$$

The acceleration a can now be determined to be

$$a = \frac{M_2 g - M_1 g(\sin\theta + \mu_k \cos\theta)}{M_1 + M_2} \tag{47}$$

If block 1 is moving down the incline, the friction force is pointed up the incline. A positive acceleration of block 1 corresponds to block 2 moving up. The net force on block 2 is equal to

$$F_2 = T - M_2 g = M_2 a \tag{48}$$

The net force on block 1 is equal to

$$F_1 = M_1 g \sin\theta - T - f_k = M_1 g \sin\theta - T - \mu_k M_1 g \cos\theta = M_1 g(\sin\theta - \mu_k \cos\theta) - T = M_1 a \tag{49}$$

Adding both equations we can eliminate T and determine a:

$$M_1 g(\sin\theta - \mu_k \cos\theta) - M_2 g = (M_1 + M_2)a \tag{50}$$

The acceleration a can now be determined to be

$$a = \frac{M_1 g(\sin\theta - \mu_k \cos\theta) - M_2 g}{M_1 + M_2} \tag{51}$$