Physics 121 Review
Midterm Exam # 3

April 23, 2026

Frank L. H. Wolfs Department of Physics and Astronomy, University of Rochester, Slide 1

1

Surviving Phy 121 Exams.

- Time your work:
  - Exam has 10 MC + 3 analytical questions.
  - Work 15 minutes on the MC questions.
  - Work 15 minutes on each of the analytical questions (45 minutes total).
  - You now have 20 minutes left to finish those questions you did not finish in the first 15 minutes.
- Write neatly:
  - You cannot earn credit if we cannot read what you wrote!
  - You cannot earn credit if we cannot read your name (list first and last name, and student ID).
- Write enough so that we can see your line of thought – you cannot earn credit for what you are thinking!

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2

Surviving Phy 121 Exams.

- Most solutions should start with a diagram, showing all forces (direction and approximate magnitude) and dimensions. All forces and dimensions should be labeled with the variables that will be used in your solution. You also need to define the coordinate system you will be using,
- Indicate what variables are known and what variables are unknown.
- Indicate which variable(s) needs to be determined.
- Indicate the principle(s) that you use to solve the problem.
- If you make any approximations, indicate them.
- Check your units!

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3

Preparing for Exam # 2

- Take the practice exam as if it was a real exam: take 80 minutes to complete it. Compare your work to the posted solutions to help you focus on specific areas.
- There will be extra office hours on Sunday 4/26 and Monday 4/27:
  - Cole Jerum: 2 pm - 4 pm, Sunday 4/26, POA.
  - Thomas Qian: 7 pm - 9 pm, Sunday 4/26, POA.
  - Justin Kenneally: 10 am - 12 pm, Monday 4/27, POA.
  - Catherine Lei: 12.30 pm - 2.30 pm, Monday 4/27, POA.
- The workshops on Monday 4/27 should be considered Q&A sessions for Exam # 3. Anyone can attend any of the workshops on Monday 4/27.

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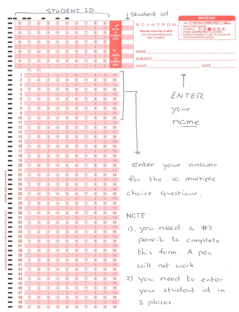
Preparing for Exam # 3

- You need a number 2 pencil to complete the multiple-choice part of the exam.
- You need to know your student ID #.
- You do NOT need a calculator on the exam.
- The exam starts at 8 am and ends at 9.20 am.
- If you are late to start, you will have less time to finish.
- No one can leave before 8.45 am. No one can start after 8.45 am.



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Lessons from Exam # 2.

- Questions 11 and 12 must be answered in Book 1.
- Question 13 must be answered in Book 2.
- By putting a 1 or 2 on the front of the booklet, you define which one is Book 1 and which one is Book 2.
- If you answer Question 13 in Book 1, it will not be graded.
- Answers that are not motivated will not receive any credit, even if correct.
- Write neatly!!!

Problems 1 and 10 must be answered on the scantron form. **Problems 11 and 12 must be answered in exam booklet 1. Problem 13 must be answered in exam booklet 2.** The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. **Answers that are not motivated will not receive any credit, even if correct.**

At the end of the exam, you must hand in your exam, the scantron form, the blue exam booklets, and the equation sheet. **All items must be clearly labeled with your name, your student ID number, and the day/time/location of your workshop. If any of these items are missing, we will not grade your exam, and you will receive a score of 0 points.**

Front cover exam

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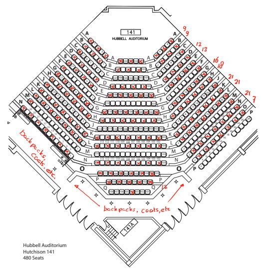
Exam Location: Hubbell.

- You can not enter Hubbel before 7.45 am. You can only sit at locations where there is a brown envelope located. Your backpack, coat, and books need to be left at the entrance of Hubbel.
- You will only need your pen, a number 2 pencil, and an eraser. Being awake might also help!



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Warning.

- I cannot cover everything I discussed in lectures 15 – 21 in this review.
- If I skip over certain topics, it does not mean you should not understand that material.
- Your TAs will not see the exam until you see it.
- **There will be lecture at 12.30 pm on Tuesday 4/28.**

8

Review Physics 121 Midterm Exam # 3

- Main topics covered on Exam # 3:
  - Rotational Motion (Chapters 10-11):
    - Rotational Variables.
    - Angular Momentum.
  - Equilibrium (Chapter 12):
    - Conditions for Equilibrium.
  - Harmonic Motion (Chapter 14):
    - Properties of Simple Harmonic Motion
    - Requirements for Simple Harmonic Motion
    - Damped Harmonic Motion
    - Driven Harmonic Motion

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Chapter 10

Rotational Motion.

Excluded Sections:
10.10

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10

Rotational Motion

- There are many similarities between linear and rotational motion.
- If you really understand linear motion, understanding rotational motion should be easy.
- The concepts of moment of inertia, torque, and angular momentum are defined such as to preserve the similarities between linear and rotational motion.
- I will start this review with focusing on a detailed comparison between linear and rotational motion.

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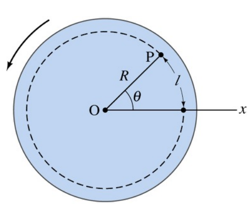
11

Rotational Motion. Variables

- In our discussion of rotational motion we focused on the rotation of **rigid** objects around a **fixed** axis.
- The variables that are used to describe this type of motion are similar to those we use to describe linear motion:
  - Angular position
  - Angular velocity
  - Angular acceleration



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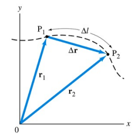
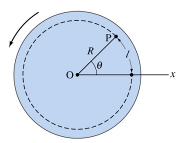
12

Rotational Variables. Position.

- In order to specify the position of a point we need to specify our reference point/axis.
- Linear position:
  - Specify the vector required to move from the origin of the coordinate system to point *P*.
  - Unit: m
- Angular position:
  - Specify the rotation angle required to rotate from the the reference line to point *P*.
  - Unit: rad





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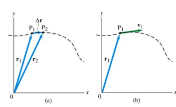
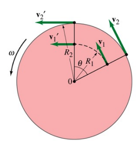
Rotational Variables. Velocity.

- Velocity is a measure of how quickly the position changes.
- Linear velocity:
  - Definition: $\vec{v} = d\vec{r}/dt$
  - Symbol: v
  - Units: m/s
- Angular velocity:
  - Definition: $\omega = d\theta/dt$
  - Symbol: ω
  - Units: rad/s





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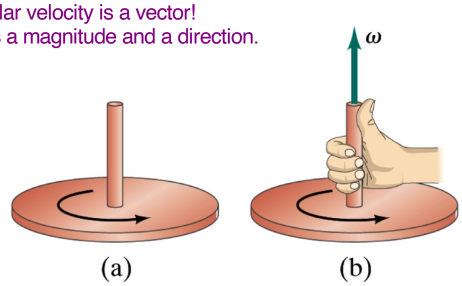
Direction of the Angular Velocity. User your Right Hand!

Angular velocity is a vector!
It has a magnitude and a direction.



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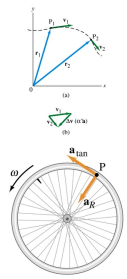
Rotational Variables. Acceleration.

- Acceleration is a measure of how quickly the velocity changes.
- Linear acceleration:
  - Definition: $\vec{a} = d\vec{v}/dt = d^2\vec{r}/dt^2$
  - Symbol: a
  - Units: m/s^2
- Angular acceleration:
  - Definition: $\alpha = d\omega/dt = d^2\theta/dt^2$
  - Symbol: α
  - Units: rad/s^2



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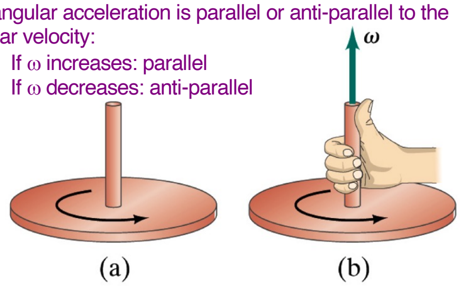
Rotational Variables. Acceleration.

The angular acceleration is parallel or anti-parallel to the angular velocity:

If ω increases: parallel

If ω decreases: anti-parallel



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Summary of Rotational Variables.

	Definition	Linear Variable
Angular Position	θ	$l = R\theta$
Angular Velocity	$\omega = \dfrac{d\theta}{dt}$	$v = R\omega$
Angular Acceleration	$\alpha_{\tan} = \dfrac{d^2\theta}{dt^2}$	$a_{\tan} = R\alpha_{\tan}$

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Equations of Rotational Motion. Constant Acceleration.

	Rotationional Motion	Linear Motion
Acceleration	$\alpha(t) = \alpha$	$a(t) = a$
Velocity	$\omega(t) = \omega_0 + \alpha t$	$v(t) = v_0 + at$
Position	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

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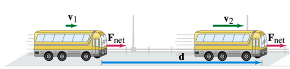
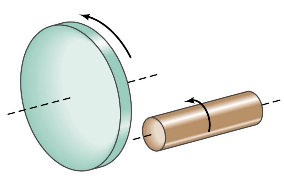
Rotational Variables. Kinetic Energy.

- The kinetic energy of an object is proportional to the square of its velocity.
- Linear kinetic energy:
  - Definition: $K = (1/2)mv^2$
  - Unit: kg m^2/s^2 or J
- Rotational kinetic energy:
  - Definition: $K = (1/2)I\omega^2$
  - Unit: kg m^2/s^2 or J
  - I is the moment of inertia of the mass distribution.





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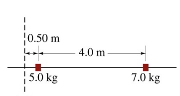
The moment of inertia. Calculating *I*.

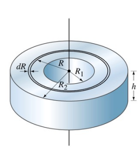
- The moment of inertia of an objects depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$





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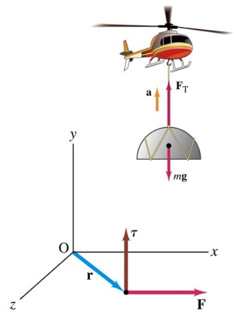
Rotational Variables. Force and Torque.

- Linear motion:
  - The linear acceleration is proportional to the applied force:
    $\vec{F} = m\vec{a}$
  - Unit: N
- Rotational motion:
  - The angular acceleration is proportional to the torque:
    $\vec{\tau} = I\vec{\alpha}$
    where I is the momentum of inertia of the system.
  - The torque $\vec{\tau}$ is defined as the vector product
    $\vec{\tau} = \vec{r} \times \vec{F}.$
  - Unit: Nm



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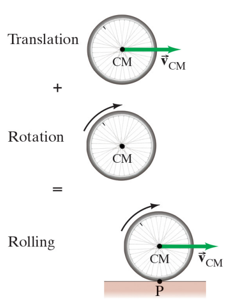
Rolling motion.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
  - Translational kinetic energy:
    $\frac{1}{2}Mv_{cm}^2$
  - Rotational kinetic energy:
    $\frac{1}{2}I_{cm}\omega^2$
- Assuming the wheel does not slip:

$$\omega = \frac{v}{R}$$



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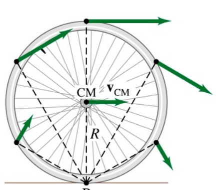
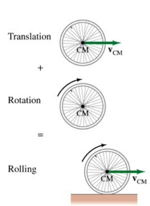
Rolling motion causes much confusion!





Two views of rolling motion: 1) Pure rotation around the instantaneous axis or 2) rotation and translation.

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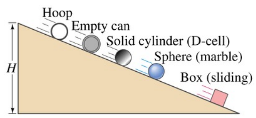
Rolling motion. Effect of Moment of Inertia on Motion.

- Initial mechanical energy:

$$E_i = mgH$$

- Final mechanical energy:

$$E_f = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$



- Assuming no slipping, we can rewrite the final mechanical energy as

$$E_f = \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right)v_{cm}^2$$

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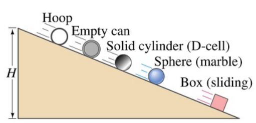
Rolling motion. Effect of Moment of Inertia on Motion.

- Conservation of energy implies:

$$\frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right)v_{cm}^2 = mgH$$

or

$$\frac{1}{2}\left(1 + \frac{I_{cm}}{mR^2}\right)v_{cm}^2 = gH$$



- The smaller I_{cm}, the larger v_{cm} at the bottom of the incline.
- Example:
  Ring: $I_{cm} = MR^2$, $v_{cm} = \sqrt{gH}$
  Disk: $I_{cm} = \frac{1}{2}MR^2$, $v_{cm} = \sqrt{\frac{4}{3}gH}$

The smaller I_{cm}, the larger v_{cm} at the bottom of the incline.

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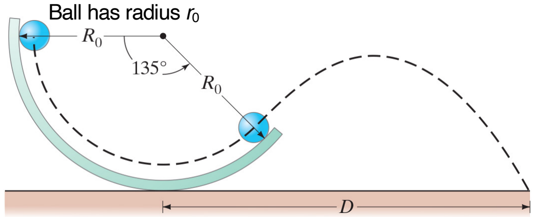
Example Problem. Problem 10.76.

Ball has radius r_0



What is the speed of the ball at the bottom of the track?

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Any Questions about Chapter 10?

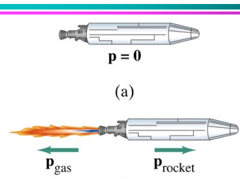
Chapter 11

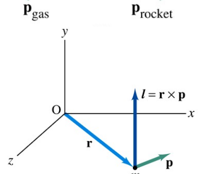
Angular Momentum.

Excluded Sections:
11-8 and 11-9.

Rotational Variables. Linear and Angular Momentum.

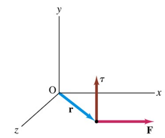
- Linear Momentum:
  - Defined: $\vec{p} = m\vec{v}$.
  - Units: kg m/s
  - Total linear momentum is conserved if the net external force is 0 N.
- Angular Momentum:
  - Defined: $\vec{L} = \vec{r} \; x \; \vec{p}$.
  - For rigid object: $\vec{L} = I\vec{\omega}$.
  - Unit: kg m^2/s
  - Total angular momentum is conserved is the net external torque is 0 Nm.

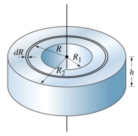


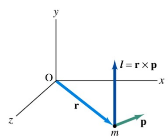


Rotational Variables. Make sure you know your references!

- The moment of inertia is calculated with respect to a specific rotation axis.
- The torque and angular momentum are calculated with respect to a specific reference point.







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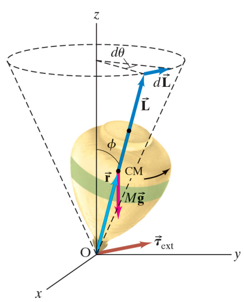
Conservation of angular momentum.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\Sigma\vec{\tau} = \frac{d\vec{L}}{dt}$$

is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).
- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).



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Precession.

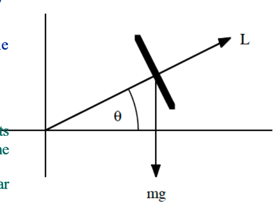
- The external torque is equal to

$$|\vec{\tau}| = |\vec{r}\times\vec{F}| = rF\sin\left(\frac{\pi}{2} + \theta\right) = mgr\cos\theta$$

- The external torque causes a change in the angular momentum:

$$d\vec{L} = \vec{\tau}dt$$

- Thus:
  - The change in the angular momentum points in the same direction as the direction of the torque.
  - The direction of the torque is perpendicular to the direction of the angular momentum.
  - The torque will thus change the direction of $\vec{L}$ but not its magnitude.



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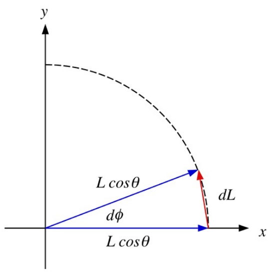
Precession.

- The effect of the torque can be visualized by looking at the motion of the projection of the angular momentum in the *xy* plane.
- The angle of rotation of the projection of the angular momentum vector when the angular momentum changes by dL is equal to

$$d\phi = \frac{dL}{L\cos\theta} = \frac{mgr\cos\theta dt}{L\cos\theta} = \frac{mgrdt}{L}$$



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Precession.

- Since the projection of the angular momentum during the time interval dt rotates by an angle $d\phi$, we can calculate the rate of precession:

$$\Omega = \frac{d\phi}{dt} = \frac{mgr}{L} = \frac{mgr}{I\omega}$$

- We conclude the following:
  - The rate of precessions does not depend on the angle θ.
  - The rate of precession decreases when the angular momentum increases.

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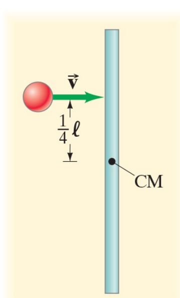
Example Problem.
Problem 11.52.

- Rod on frictionless surface of mass M and length ℓ.
- Ball of clay of mass m sticks to the rod.
- Determine its translational and rotational motion.
- Notes:
  - Determine what our reference point should be.
  - Apply conservation of angular and linear momentum.



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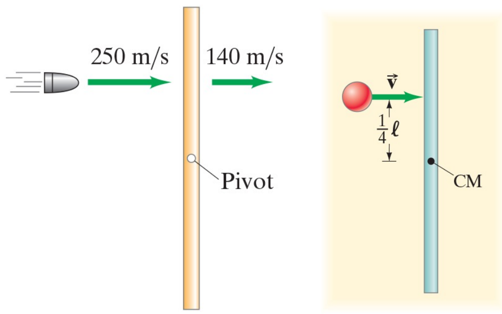
What is the difference?



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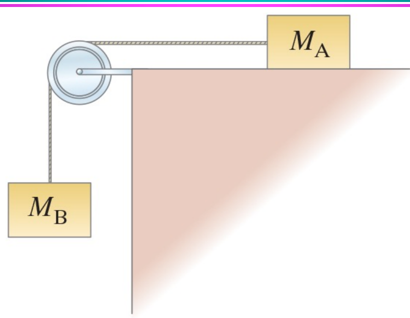
When pulleys have mass, $T_A \neq T_B$.



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Any Questions about Chapter 11?

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Chapter 12

Equilibrium.

Excluded Sections:
12-6, 12-7, and 12.8.

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Equilibrium.

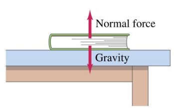
- An object is in equilibrium is the following conditions are met:

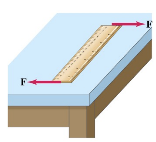
  Net force = 0 N (**first** condition for equilibrium)

  and

  Net torque = 0 Nm (**second** condition for equilibrium)

- Note: both conditions must be satisfied. Even if the net force is 0 N, the system can start to rotate if the net torque is not equal to 0 Nm.





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Static Equilibrium.

- What happens when the net force is equal to 0 N?
  - P = constant
- What happens when the net torque is equal to 0 Nm?
  - L = constant
- We conclude that an object in equilibrium can still move (with constant linear velocity) and rotate (with constant angular velocity).
- Conditions for **static** equilibrium:
  - P = 0 kg m/s
  - L = 0 kg m^2/s

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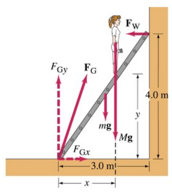
Equilibrium. Summary of conditions.

- Equilibrium in 3D:

$$\sum F_x = 0 \qquad \sum \tau_x = 0$$
$$\sum F_y = 0 \quad \text{and} \quad \sum \tau_y = 0$$
$$\sum F_z = 0 \qquad \sum \tau_z = 0$$

- Equilibrium in 2D:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau_z = 0$$



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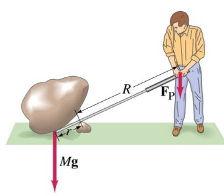
Equilibrium. Be sure to include all forces!!!

- When evaluating conditions for equilibrium, you need to make sure to include all forces acting on the system.
- In the system shown in the Figure, there are more forces acting on the system than the forces indicated. For example, there should be an upward force to balance the downward forces.
- Of course, the problem is how to apply the equilibrium conditions correctly.



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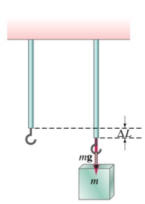
Stress and Strain. The Effect of Applied Forces.

- When we apply a force to an object that is kept fixed at one end, its dimensions can change.
- If the force is below a maximum value, the change in dimension is proportional to the applied force. This is called Hooke's law:

$$F = k\Delta L$$

- This force region is called the elastic region.



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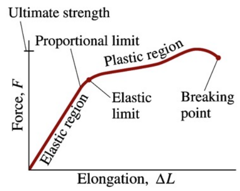
Stress and Strain.
The Effect of Applied Forces.

- When the applied force increases beyond the elastic limit, the material enters the plastic region.
- The elongation of the material depends not only on the applied force F, but also on the type of material, its length, and its cross-sectional area.
- In the plastic region, the material does not return to its original shape (length) when the applied force is removed.



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Stress and strain.
The effect of applied forces.

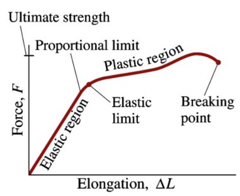
- The elongation ΔL can be specified as follows:

$$\Delta L = \frac{1}{E}\frac{F}{A}L_0$$

where

L_0 = original length
A = cross sectional area
E = Young's modulus

- **Stress** is defined as the force per unit area (= F/A).
- **Strain** is defined as the fractional change in length ($\Delta L_0/L_0$).



Note: the ratio of stress to strain is equal to the Young's Modulus.

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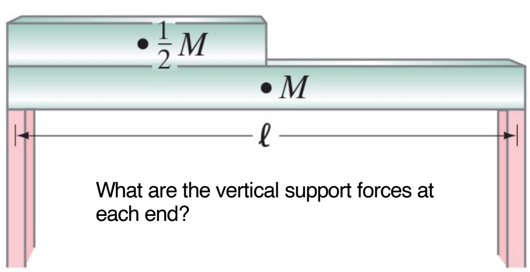
Example Problem.
Problem 12.16.



What are the vertical support forces at each end?

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Any Questions about Chapter 12?

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Chapter 14

Simple Harmonic Motion.

Excluded Sections:
none

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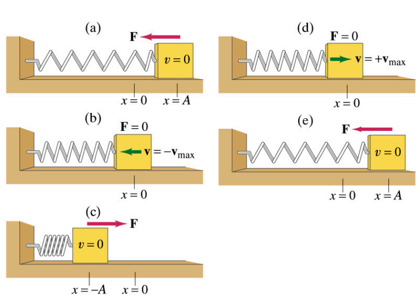
Harmonic motion.
Motion that repeats itself at regular intervals.



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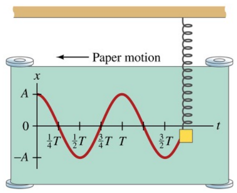
Simple harmonic motion.

Phase Constant

Amplitude

$$x(t) = A\cos(\omega t + \phi)$$

Angular Frequency



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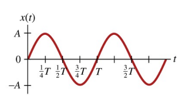
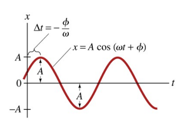
Simple Harmonic Motion.

- Other variables frequently used to describe simple harmonic motion:
  - The period T: the time required to complete one oscillation. The period T is equal to $2\pi/\omega$.
  - The frequency of the oscillation is the number of oscillations carried out per second:

    $\nu = 1/T$

    The unit of frequency is the Hertz (Hz). Per definition, 1 Hz = 1 s^{-1}.





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Simple harmonic motion. What forces are required?

- Using Newton's second law we can determine the force responsible for the harmonic motion:

$$F = ma = -m\omega^2 x$$

- A good example of a force that produces simple harmonic motion is the spring force: $F = -kx$. The angular frequency depends on both the spring constant k and the mass m:

$$\omega = \sqrt{\frac{k}{m}}$$

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Simple Harmonic Motion (SHM). The equation of motion.

- All examples of SHM were derived from he following equation of motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

- The most general solution to the equation is

$$x(t) = A\cos(\omega t + \alpha) + B\sin(\omega t + \beta)$$

- SHM will occur when $A = B$, $A = 0$, or $B = 0$.

Damped Harmonic Motion.

- Consider what happens when in addition to the restoring force a damping force (such as the drag force) is acting on the system:

$$F = -kx - b\frac{dx}{dt}$$

- The equation of motion is now given by:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

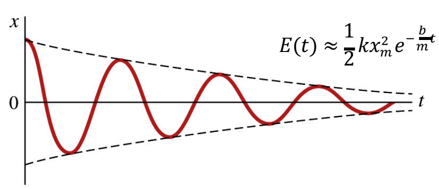
- The solution to the equation of motion is thus given by

$$x(t) \approx x_m e^{-\frac{b}{2m}t} e^{it\sqrt{\frac{k}{m}}}$$

Damped Harmonic Motion.

$$x(t) \approx x_m e^{-\frac{b}{2m}t} e^{it\sqrt{\frac{k}{m}}}$$

$$E(t) \approx \frac{1}{2}kx_m^2 e^{-\frac{b}{m}t}$$



The general solution contains a SHM term, with an amplitude that decreases as function of time

Driven Harmonic Motion.

- Consider what happens when we apply a time-dependent force $F(t)$ to a system that normally would carry out SHM with an angular frequency ω_0.
- Assume the external force $F(t) = mF_0\sin(\omega t)$. The equation of motion can now be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x + F_0 \sin(\omega t)$$

- The steady state motion of this system will be harmonic motion with an angular frequency equal to the angular frequency of the driving force.
- The amplitude of the motion is equal to

$$A = \frac{F_0}{(\omega_0^2 - \omega^2)}$$

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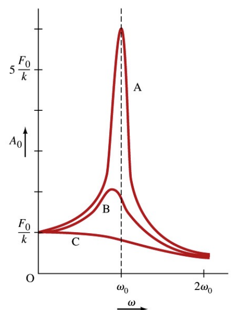
Driven Harmonic Motion.

- If the driving force has a frequency close to the natural frequency of the system, the resulting amplitudes can be very large even for small driving amplitudes. The system is said to be in resonance.
- In realistic systems, there will also be a damping force. Whether or not resonance behavior will be observed will depend on the strength of the damping term.



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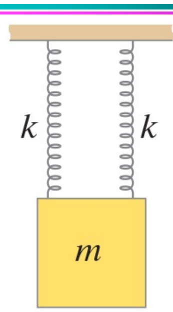
Example Problem. Problem 14.21.

- What will be the frequency of the vertical oscillations?



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Example Problem.
Problem 14.95.

Use shell theorem to show that the net force on mass *m* is proportional to *r*.

Simple harmonic motion!

Any Questions about Chapter 14?

Final Remarks

- The hardest part of each problem is recognizing the approach to take. Different approaches may lead to the same answer, but can differ greatly in difficulty.
- A suggestion:
  - Look at the end of chapter problems. There is only a limited number of types of question one can ask.
  - But ……. Since the questions are grouped by section, you know already what approach to use based on the section to which the problems are assigned.
  - Some students benefit from copying the questions, cutting them out, writing the chapter/section numbers on the back, mixing them up, and then reading through them and determining what approach you would take if you would see that question on the exam (compare it with the focus of the section to which the problem was assigned).

Final Remarks

- You will only need your pen, a pencil, and an eraser. Being awake might also help!
- The TAs will have extra office hours on Sunday and Monday. Please go and see them if you need to resolve any last-minute questions.
- The exam will start at 8 am and end at 9.20 am. If you show up late you will just have less time to finish. Over the years I have heard every excuse possible for being late, but I have never heard one that I accepted.

Good luck preparing for the exam.

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