

Physics 121.
Lecture 18.



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Physics 121.
Lecture 18.

- Course Information
- Topics to be discussed today:
 - Review of Angular Momentum
 - Conservation of Angular Momentum
 - Precession

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Physics 121.
March 27, 2008.

- Homework set # 6 is now available and is due on Saturday April 4 at 8.30 am.
- Homework set # 7 is also available and is due on Saturday April 11 at 8.30 am.
- Homework set # 7 has two components:
 - WeBWork (75%)
 - Video analysis (25%)
- Make sure you pick up the results of exam # 2 during workshop this week.

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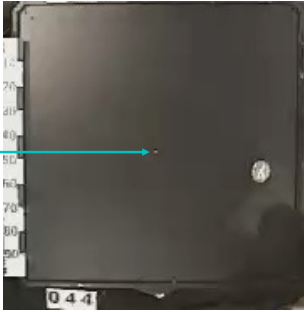
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Video analysis.

Is angular momentum conserved?

Ruler →

Origin →



Mass puck is 33.6 g.

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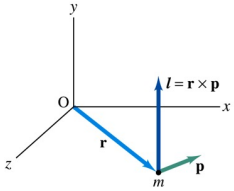
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Angular momentum.

A quick review.

- We have seen many similarities between the way in which we describe linear and rotational motion.
- Our treatment of these types of motion are similar if we recognize the following equivalence:

<u>linear</u>	<u>rotational</u>
mass m	moment I
force $\vec{F}$	torque $\vec{\tau} = \vec{r} \times \vec{F}$
- What is the equivalent to linear momentum?
 Answer: **angular momentum.**



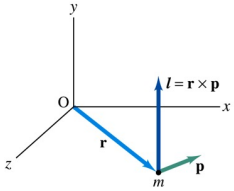
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Angular momentum.

A quick review.

- The angular momentum is defined as the vector product between the position vector and the linear momentum.
- Note:
 - Compare this definition with the definition of the torque.
 - Angular momentum is a vector.
 - The unit of angular momentum is $\text{kg m}^2/\text{s}$.
 - The angular momentum depends on both the magnitude and the direction of the position and linear momentum vectors.
 - Under certain circumstances the angular momentum of a system is conserved!



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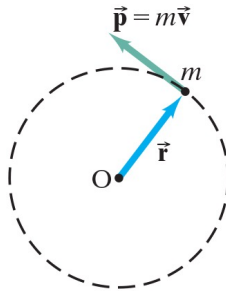
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Angular momentum. A quick review.

- Consider an object carrying out circular motion.
- For this type of motion, the position vector will be perpendicular to the momentum vector.
- The magnitude of the angular momentum is equal to the product of the magnitude of the radius $\vec{r}$ and the linear momentum $\vec{p}$:

$$L = mvr = mr^2 \frac{v}{r} = I\omega$$

- Note: compare this with the linear momentum $p = mv$!



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Conservation of angular momentum. A quick review.

- Consider the change in the angular momentum of a particle:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = m \left(\vec{r} \times \frac{d\vec{v}}{dt} + \frac{d\vec{r}}{dt} \times \vec{v} \right) = m(\vec{r} \times \vec{a} + \vec{v} \times \vec{v})$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times m\vec{a} = \vec{r} \times \sum \vec{F} = \sum \vec{\tau}$$

- Consider what happens when the net torque is equal to 0:

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow L = \text{constant}$$

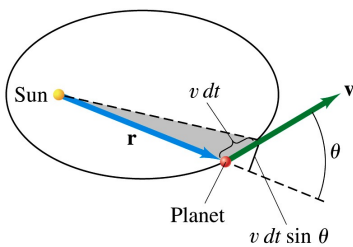
- When we take the sum of all torques, the torques due to the internal forces cancel and the sum is equal to torque due to all external forces.
- Note: notice again the similarities between linear and rotational motion.

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Conservation of angular momentum. Consequences.



$$|\vec{L}| = rmv \sin \theta = m \frac{rv dt \sin \theta}{dt} = 2m \frac{dA}{dt} = \text{constant}$$

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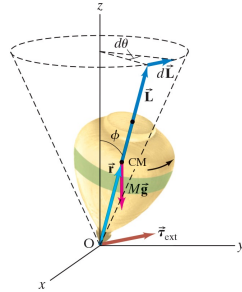
Conservation of angular momentum. A quick review.

- The connection between the angular momentum $\vec{L}$ and the torque $\vec{\tau}$

$$\Sigma \vec{\tau} = \frac{d\vec{L}}{dt}$$

is only true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the same reference point (which is at rest in an inertial reference frame).

- The relation is also true if $\vec{L}$ and $\vec{\tau}$ are calculated with respect to the center of mass of the object (note: center of mass can accelerate).



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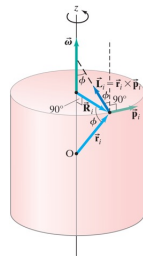
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The angular momentum of rotating rigid objects.

- Consider a rigid object rotating around the z axis.
- The magnitude of the angular momentum of a part of a small section of the object is equal to

$$l_i = r_i(m_i v_i)$$

- Due to the symmetry of the object, we expect that the angular momentum of the object will be directed on the z axis. Thus, we only need to consider the z component of this angular momentum.



**Note the direction of l_i !!!!
(perpendicular to r_i and p_i)**

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The angular momentum of rotating rigid objects.

- The z component of the angular momentum is

$$L_{i,z} = l_i \sin \theta = (r_i \sin \theta) m_i v_i = R_i m_i v_i$$

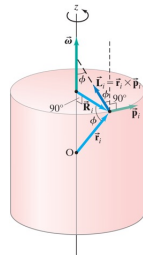
- The total angular momentum of the rotating object is the sum of the angular momenta of the individual components:

$$L_z = \Sigma L_{i,z} = \Sigma R_i (m_i \omega R_i) = \omega \Sigma m_i R_i^2$$

- The total angular momentum is thus equal to

$$L_z = I \omega$$

**Note the direction of l_i !!!!
(perpendicular to r_i and p_i)**



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Conservation of angular momentum. Sample problem.

- The moment inertia of a collapsing spinning star changes to one-third of its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?
- When the star collapses, it is compressed, and its moment of inertia decreases. In this particle case, the reduction is a factor of 3:

$$I_f = \frac{1}{3}I_i$$

- The forces responsible for the collapse are internal forces, and angular momentum is conserved.

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Conservation of angular momentum. Sample problem.

- The initial kinetic energy of the star can be expressed in terms of its initial angular momentum:

$$K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\frac{L_i^2}{I_i}$$

- The final kinetic energy of the star can also be expressed in terms of its angular momentum:

$$K_f = \frac{1}{2}\frac{L_f^2}{I_f} = \frac{1}{2}\frac{L_i^2}{\frac{1}{3}I_i} = 3K_i$$

- Note: **the kinetic energy increased!** Where does this energy come from?

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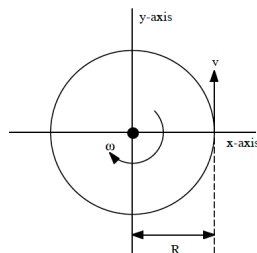
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Conservation of angular momentum. Sample problem.

A cockroach with mass m runs counterclockwise around the rim of a lazy Susan (a circular dish mounted on a vertical axle) of radius R and rotational inertia I with frictionless bearings. The cockroach's speed (with respect to the earth) is v , whereas the lazy Susan turns clockwise with angular speed ω ($\omega < 0$). The cockroach finds a bread crumb on the rim and, of course, stops.

- What is the angular speed of the lazy Susan after the cockroach stops?
- Is mechanical energy conserved?



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Conservation of angular momentum. Sample problem.

- The initial angular momentum of the cockroach is

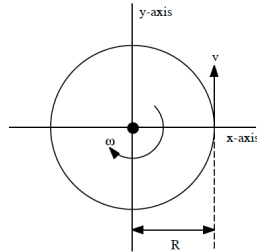
$$\vec{L}_c = \vec{r} \times \vec{p} = Rmv \hat{z}$$

- The initial angular momentum of the lazy Susan is

$$\vec{L}_d = I\omega \hat{z}$$

- The total initial angular momentum is thus equal to

$$\vec{L} = \vec{L}_c + \vec{L}_d = (Rmv + I\omega) \hat{z}$$



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Conservation of angular momentum. Sample problem.

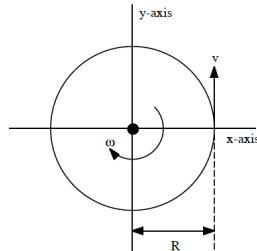
- When the cockroach stops, it will move in the same way as the rim of the lazy Susan. The forces that bring the cockroach to a halt are internal forces, and angular momentum is thus conserved.

- The moment of inertia of the lazy Susan + cockroach is equal to

$$I_f = I + mR^2$$

- The final angular velocity of the system is thus equal to

$$\omega_f = \frac{L_f}{I_f} = \frac{Rmv + I\omega}{I + mR^2}$$



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Conservation of angular momentum. Sample problem.

- The initial kinetic energy of the system is equal to

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Cockroach **Lazy Susan**

- The final kinetic energy of the system is equal to

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(I + mR^2)\left(\frac{Rmv + I\omega}{I + mR^2}\right)^2 = \frac{1}{2}\frac{(Rmv + I\omega)^2}{I + mR^2}$$

- The change in the kinetic energy is thus equal to

$$\Delta K = K_f - K_i = -\frac{1}{2}\frac{mI}{I + mR^2}(v - R\omega)^2$$

Loss of Kinetic Energy!!

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4 Minute 32 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 32 second intermission.
- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.



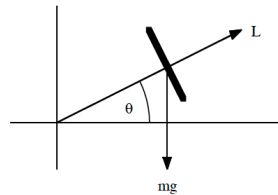
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Precession.

- Consider a rotating rigid object spinning around its symmetry axis.
- The object carries a certain angular momentum L .
- Consider what will happen when the object is balanced on the tip of its axis (which makes an angle θ with the horizontal plane).
- The gravitational force, which is an external force, will generate a torque with respect of the tip of the axis.



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Precession.

- The external torque is equal to

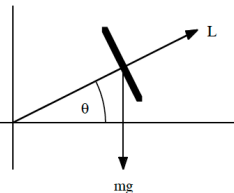
$$|\vec{\tau}| = |\vec{r} \times \vec{F}| = rF \sin\left(\frac{\pi}{2} + \theta\right) = mgr \cos\theta$$

- The external torque causes a change in the angular momentum:

$$d\vec{L} = \vec{\tau} dt$$

† Thus:

- The change in the angular momentum points in the same direction as the direction of the torque.
- The direction of the torque is perpendicular to the direction of the angular momentum.
- The torque will thus change the direction of $\vec{L}$ but not its magnitude.



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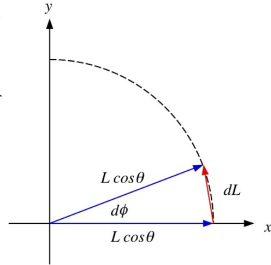
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Precession.

- The effect of the torque can be visualized by looking at the motion of the projection of the angular momentum in the xy plane.
- The angle of rotation of the projection of the angular momentum vector when the angular momentum changes by dL is equal to

$$d\phi = \frac{dL}{L \cos \theta} = \frac{mgr \cos \theta dt}{L \cos \theta} = \frac{mgr dt}{L}$$



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Precession.

- Since the projection of the angular momentum during the time interval dt rotates by an angle $d\phi$, we can calculate the rate of precession:

$$\Omega = \frac{d\phi}{dt} = \frac{mgr}{L} = \frac{mgr}{I\omega}$$

- We conclude the following:
 - The rate of precessions does not depend on the angle θ .
 - The rate of precession decreases when the angular momentum increases.



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Done for today! Next week we stop moving and focus on equilibrium.



Spirit Pan from Bonneville Crater's Edge
Credit: [Mars Exploration Rover Mission](#), [JPL](#), [NASA](#)

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