Note: The numerical values in these solutions are correct for my version of the assignment. Your numbers are different.

WebWork Problem 1

The force F is given by $F(x) = 6.4x^2 + 4.3x$. In order to determine the potential energy at $x = 3.0$ m, we need to calculate the path integral of F along the path that connects $x = 0$ m with $x = 3.0$ m. In the region between $x = 0$ m and $x = 3.0$ m, the force is positive and force and displacement are pointing in the same direction. The work done by the force is thus positive. Consider a small path segment at x: dx. The work done by F when we move from x to $x+dx$ is equal to $dW = (6.4x^2+4.3x)dx$. The potential energy at $x = 3.0$ m is thus equal to

$$U(3) = -\int_{x=0}^{x=3}(6.4x^2 + 4.3x)dx = -\left(\frac{6.4}{3}x^3 + \frac{4.3}{2}x^2\right)\Bigg|_0^3 = -77J \qquad (1)$$

WebWork Problem 2
The initial mechanical energy of an object of mass m fired from the surface of the earth with a velocity v_i is equal to

$$E_i = \frac{1}{2} m v_i^2 - G \frac{m M_E}{R_E} \tag{2}$$

At an infinite distance, the potential energy will be zero, and the mechanical energy will thus be equal to the kinetic energy of the object:

$$E_f = \frac{1}{2} m v_f^2 \tag{3}$$

Since mechanical energy is conserved, we require that

$$\frac{1}{2} m v_i^2 - G \frac{m M_E}{R_E} = \frac{1}{2} m v_f^2 \tag{4}$$

or

$$v_i = \sqrt{v_f^2 + 2G \frac{M_E}{R_e}} \tag{5}$$

WebWork Problem 3

We will take the top of the spring (in its rest position) to be the position where the potential energy is equal to 0 J. When the elevator is at rest a distance d above the spring, it will only have potential energy ($E = mgd$). When the cable snaps, the elevator will move down, but its downwards motion is slowed down by the clamps on the guide rails that provide a constant frictional force f. The work done by the friction force up to the point that the elevator reaches the spring is $-fd$. The mechanical energy of the system at this point will thus be equal to $mgd-fd$. At this position, all the mechanical energy is in the form of kinetic energy of the elevator.

Consider what happens when the spring is compressed by a distance x. Assuming that this is the maximum compression (that is, the elevator has come to rest), we know that the total mechanical energy of the system is $-mgx+(1/2)kx^2$. During the compression, the work done by the friction force is $-fx$, and the total mechanical energy must therefore also be equal to $mgd-fd-fx$. We thus require that

$$\frac{1}{2}kx^2 - mgx = mgd - fd - fx \tag{6}$$

This equation can be rewritten as

$$\frac{1}{2}kx^2 + (f - mg)x + (f - mg)d = 0 \tag{7}$$

which can be solved for x:

$$x = \frac{-(f-mg) \pm \sqrt{(f-mg)^2 - 4(f-mg)d\frac{1}{2}k}}{k} = \frac{mg-f}{k}\left(1 \pm \sqrt{1 + 2\frac{kd}{mg-f}}\right) \tag{8}$$

WebWork Problem 4
Suppose the velocity of the bullet is v_i. The initial momentum of the system is

$$p_i = m_1 v_i \tag{9}$$

The final velocity of the block plus bullet is v_f. The final momentum is

$$p_f = (m_1 + m_2) v_f \tag{10}$$

Conservation of linear momentum implies

$$m_1 v_i = (m_1 + m_2) v_f \tag{11}$$

or

$$v_f = \frac{m_1}{m_1 + m_2} v_i \tag{12}$$

The initial kinetic energy of the block plus bullet is

$$K_f = \frac{1}{2}(m_1 + m_2) v_f^2 \tag{13}$$

Since it is assumed that the block plus bullet swing frictionless, the total mechanical energy of the block plus bullet must be conserved. At its highest point, the mechanical energy of the block plus bullet is equal to

$$E = (m_1 + m_2) g h \tag{14}$$

Conservation of mechanical energy implies that

$$\frac{1}{2}(m_1 + m_2) v_f^2 = (m_1 + m_2) g h \tag{15}$$

or

$$v_f = \sqrt{2gh} \tag{16}$$

The velocity of the bullet can now be calculated:

$$v_i = \frac{m_1 + m_2}{m_1} v_f \tag{17}$$

We find that $v_i = 825$ m/s.

WebWork Problem 5
This problem can be solved using the first rocket equation:

$$RU_0 = Ma_{rocket} \tag{18}$$

In order to provide a specific acceleration, we can use this equation to determine the rate of fuel consumption R:

$$R = \frac{Ma_{rocket}}{U_0} \tag{19}$$

In the first part of the problem we are calculating the rate of fuel consumption required to provide a thrust equal to the weight of the rocket:

$$RU_0 = Mg \tag{20}$$

or

$$R = \frac{Mg}{U_0} = 55.5 \text{ kg/s} \tag{21}$$

To provide an initial acceleration of 5 m/s^2, the thrust provided by the rocket engine must be $M(g+5)$. The required fuel consumption is thus equal to

$$R = \frac{M(g+5)}{U_0} = 83.9 \text{ kg/s} \tag{22}$$

WebWork Problem 6
The force exerted on the tennis ball is shown in Fig. 1.

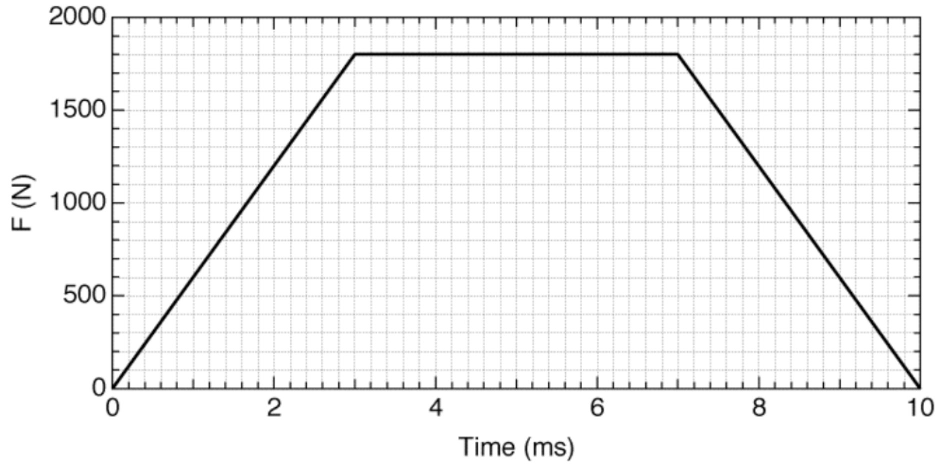

Figure 1: Force on a tennis ball.

The change in the momentum of the tennis ball is determined by the impulse of the force. The graph can be used to calculate the impulse of the force, which is the area under the force versus time curve. To determine the total impulse we consider three time periods separately:

1. 0 – 3 ms: impulse = 0.003 * 900 = 2.7 Ns (note: the average force during this period is 900 N).
2. 3 – 7 ms: impulse = 0.004 * 1800 = 7.2 Ns.
3. 7 – 10 ms: impulse = 0.003 * 900 = 2.7 Ns (note: average force during this period is 900 N).

The total impulse is thus equal to 2.7 + 7.2 + 2.7 Ns = 12.6 Ns.
The change in the momentum of the tennis ball is equal to the impulse J:

$$p_f - p_i = m(v_f - v_i) = J \tag{23}$$

The final velocity of the tennis ball can now be calculated:

$$v_f = v_i + \frac{J}{m} \tag{24}$$

Note: to get the correct answer you must realize that when the tennis ball collides with the wall, the velocity changes sign. If the force exerted by the wall is positive, as is indicated in the Figure, then initial velocity of the tennis ball must be negative!

WebWork Problem 7
The two-dimensional collision studied in this problem is shown schematically in Fig. 2.

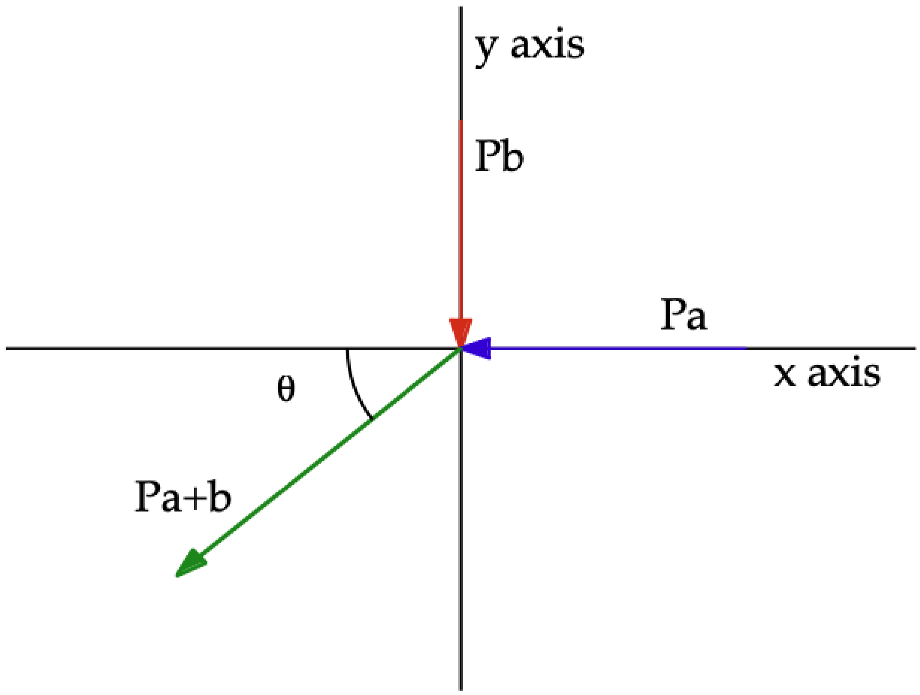

Figure 2: A two-dimensional collision.

The total linear momentum before the collision is equal to

$$\vec{p}_i = -m_A v_A \hat{x} - m_B v_B \hat{y} \tag{25}$$

Note: the minus signs indicate the direction of motion. After the collision, both cars stick together and move as one object. Since linear momentum is conserved we know that

$$\vec{p}_f = (m_A + m_B)(v_{x,f}\hat{x} + v_{y,f}\hat{y}) = \vec{p}_i = -m_A v_A \hat{x} - m_B v_B \hat{y} \tag{26}$$

Two vectors are only equal if their components are equal. We thus require that

$$(m_A + m_B)v_{x,f} = -m_A v_A \tag{27}$$

and

$$(m_A + m_B)v_{y,f} = -m_B v_B \tag{28}$$

The final velocity of the combined system is thus equal to

$$\vec{v}_f = v_{x,f}\hat{x} + v_{y,f}\hat{y} = -\frac{m_A v_A}{m_A + m_B}\hat{x} - \frac{m_B v_B}{m_A + m_B}\hat{y} \tag{29}$$

The final speed of the combined system is thus equal to

$$|\vec{v}_f| = \sqrt{\left(\frac{m_A v_A}{m_A + m_B}\right)^2 + \left(\frac{m_B v_B}{m_A + m_B}\right)^2} = \frac{1}{m_A + m_B}\sqrt{(m_A v_A)^2 + (m_B v_B)^2} = 35.6 \text{ m/s} \tag{30}$$

The direction of the combined system can be determined by finding the angle θ (see Fig. 2):

$$\tan\theta = \frac{v_{f,y}}{v_{f,x}} = \frac{-\frac{m_B v_B}{m_A+m_B}}{-\frac{m_A v_A}{m_A+m_B}} = \frac{m_B v_B}{m_A v_A} \tag{31}$$

The angle θ is thus equal to

$$\theta = \arctan\left(\frac{m_B v_B}{m_A v_A}\right) = 28.8^\circ \tag{32}$$

The angle with respect to East is thus $28.8^\circ + 180^\circ = 208.8^\circ$.

WebWork Problem 8
The first step to solve this problem is using conservation of mechanical energy to determine the velocity of the first pendulum just before the inelastic collision. Conservation of energy tells us that

$$m_1 g d = \frac{1}{2} m_1 v_1^2 \tag{33}$$

or

$$v_1 = \sqrt{2gd} \tag{34}$$

The total linear momentum before the collision is thus equal to

$$p_i = m_1 v_1 = m_1 \sqrt{2gd} \tag{35}$$

Since linear momentum is conserved, we can now calculate the final velocity of the combined system:

$$p_f = (m_1 + m_2) v_f = p_i = m_1 \sqrt{2gd} \tag{36}$$

or

$$v_f = \frac{m_1 \sqrt{2gd}}{m_1 + m_2} \tag{37}$$

After the collision, mechanical energy is conserved again and the system will rise a distance h. This requires that

$$\frac{1}{2}(m_1 + m_2) v_f^2 = (m_1 + m_2) g h \tag{38}$$

This equation can be used to determined h:

$$h = \frac{v_f^2}{2g} = \frac{1}{2g} \frac{(m_1 \sqrt{2gd})^2}{(m_1 + m_2)^2} = \frac{1}{2g} \frac{2gd m_1^2}{(m_1 + m_2)^2} = d \frac{m_1^2}{(m_1 + m_2)^2} = 0.207 \text{ m} \tag{39}$$

WebWork Problem 9
A two-dimensional collision is shown in Fig. 3. The figure shows the positions of the two object involved in the collision at 3-s intervals. No matter what the nature of the collision is, linear momentum must be conserved.

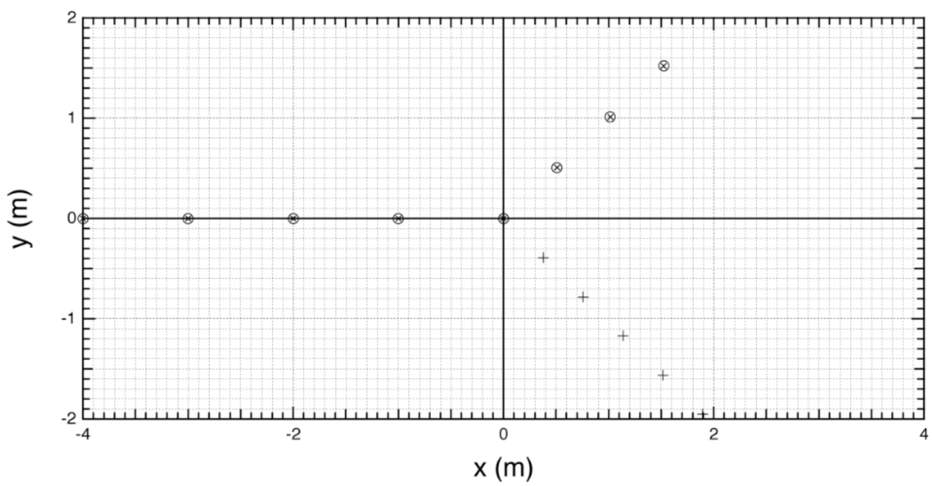

Figure 3: Two-dimensional collision.

The following analysis is correct for the Figure that was part of my assignment (see Figure above). This Figure may be different from the Figure shown on your assignment.

1. Using the data of the incoming object, we can determine its velocity to be +4/12 m/s = 0.333 m/s.
2. The total initial linear momentum of the system is thus equal to +2.33 kg m/s and is directed along the positive x axis.
3. The y component of the velocity of the incoming mass after the collision is +1/6 = 0.167 m/s. The y component of the momentum of this mass is thus 1.17 kg m/s.
4. The y component of the velocity of the unknown object after the collision is -0.8/6 m/s = -0.133 m/s. The y component of the momentum of this mass is thus –0.133M kg/ms.
5. Since the net momentum in the vertical direction must be zero, we must require that 0.133M kg m/s = 1.17 kg m/s. The mass of the unknown object is thus equal to 8.80 kg.
6. The initial kinetic energy of the incoming object is 0.5*7*0.3332 J = 0.39 J.
7. The final speed of the incoming object is 2.15/9 = 0.24 m/s. The final kinetic energy of this object is thus equal to 0.20 J.
8. The final speed of the unknown object is 2.15/12 = 0.18 m/s. The final kinetic energy of this object is thus equal to 0.14 J.

9. The total kinetic energy after the collision is thus equal to 0.34 J.
10. The loss of kinetic energy in the collision is thus equal to 0.05 J.

WebWork Problem 10
Consider the two carts. Since they are initially at rest, the initial linear momentum of the system is 0 kg m/s. The momentum of the collision must also be equal to 0 kg m/s and we must require that

$$m_x v_x + m_y v_y = 0 \tag{40}$$

or

$$v_x = -\frac{m_y}{m_x} v_y \tag{41}$$

If both carts need to hit the ends at the same time, we also must require that

$$\frac{B}{|v_x|} = \frac{A}{|v_y|} \tag{42}$$

or

$$|v_x| = \frac{B}{A}|v_y| \tag{43}$$

Comparing the two equations that relate the velocities of the carts, we conclude that

$$\frac{B}{A} = \frac{m_y}{m_x} \tag{44}$$

The mass of cart X is thus equal to

$$m_x = m_y \frac{A}{B} \tag{45}$$

For my problem, the mass of cart X must be equal to 2.53 kg. Since the cart itself has a mass of 1 kg, we need to put 1.53 kg on cart X.

Written Problem 2
The following 6 Figures show the LoggerPro analysis results for elastic and inelastic collision: