Physics 121.
Lecture 19.

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Physics 121.
Lecture 19.

- Course Information
- Topics to be discussed today:
  - Requirements for Equilibrium
  - Gravitational Equilibrium
  - Sample problems

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- Homework set # 7 is due on Saturday morning, April 11, at 8.30 am. This assignment has two components:
  - WeBWorK (75%)
  - Video analysis (25%)
- Homework set # 8 is due on Saturday morning, April 18 at 8.30 am.
- Homework sets # 9 and # 10 are due on Saturday May 2 at 8.30 am. Note: homework set # 10 is optional. I will drop the lowest homework grade when I determine the average.

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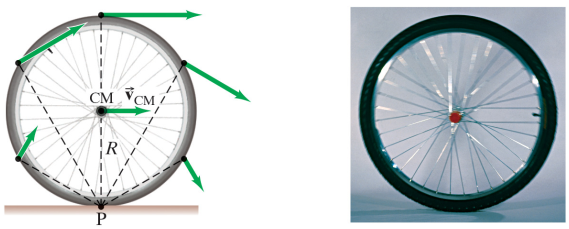
One more comment on rolling motion. Rotation around point P.



(a) (b)

Use parallel-axis theorem to determine *I*.

Equilibrium.

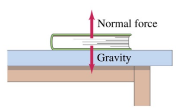
- An object is in equilibrium is the following conditions are met:

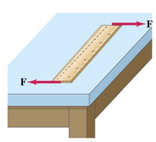
  Net force = 0 N (**first** condition for equilibrium)

  and

  Net torque = 0 Nm (**second** condition for equilibrium)

- Note: both conditions must be satisfied. Even if the net force is 0 N, the system can start to rotate if the net torque is not equal to 0 Nm.





Static Equilibrium.

- What happens when the net force is equal to 0 N?
  - P = constant
- What happens when the net torque is equal to 0 Nm?
  - L = constant
- We conclude that an object in equilibrium can still move (with constant linear velocity) and rotate (with constant angular velocity).
- Conditions for **static** equilibrium:
  - $P = 0$ kg m/s
  - $L = 0$ kg m^2/s

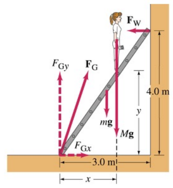
Equilibrium.
Summary of conditions.

- Equilibrium in 3D:

$$\sum F_x = 0 \quad \sum \tau_x = 0$$
$$\sum F_y = 0 \quad \text{and} \quad \sum \tau_y = 0$$
$$\sum F_z = 0 \quad \sum \tau_z = 0$$

- Equilibrium in 2D:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau_z = 0$$



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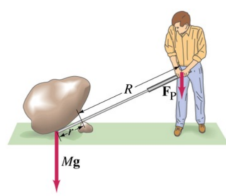
Equilibrium.
Be sure to include all forces!!!

- When evaluating conditions for equilibrium, you need to make sure to include all forces acting on the system.
- In the system shown in the Figure, there are more forces acting on the system than the forces indicated. For example, there should be an upward force to balance the downward forces.
- Of course, the problem is how to apply the equilibrium conditions correctly.



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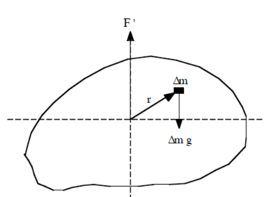
Equilibrium.
The force of gravity.

- Consider an extended rigid object that can rotate around a specific rotation point.
- If the rotation point coincides with the center-of-gravity of the object, it will be in static equilibrium in any orientation.
- What is the relation between the position of the center of mass and the position of the center of gravity?



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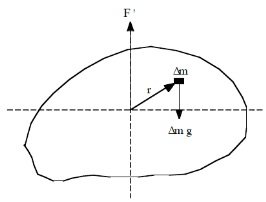
Equilibrium. The force of gravity.

- If the object is in equilibrium, the net torque and the net force acting on it must be equal to 0.
- The net force acting on the object is equal to

$$\sum\vec{F} = (F' - \sum\Delta mg)\hat{y} = (F' - Mg)\hat{y}$$

- If the net force is equal to 0 N, we must require that

$$F' = mg$$



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Equilibrium. The force of gravity.

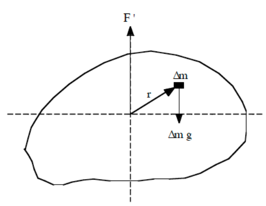
- The condition that $F' = Mg$ is not sufficient for static equilibrium. We must also require that the net torque is equal to 0 Nm.
- The net torque acting on the object is equal to

$$\sum\vec{\tau} = \sum\left(\vec{r}\times(\Delta m\vec{g})\right) = (\sum\Delta m\vec{r})\times\vec{g} \Rightarrow$$

$$\sum\vec{\tau} = M\vec{r}_{cm}\times\vec{g}$$

- If the net torque must be 0 Nm, we must require that

$$M\vec{r}_{cm}\times\vec{g} = 0$$



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Equilibrium. The force of gravity.

- The system will be in equilibrium if

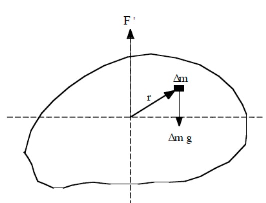
$$M\vec{r}_{cm}\times\vec{g} = 0$$

- The requires that

The center-of-gravity is located exactly below or above the rotation axis ($\vec{r}_{cm}$ parallel to vertical axis).

or

The center-of-gravity coincides with the rotation axis ($\vec{r}_{cm} = 0$).



Used to determine the location of the center-of gravity of an object.

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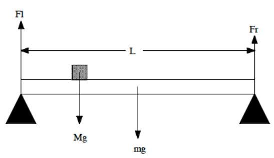
Equilibrium. Sample problem 1.

- A uniform beam of length L whose mass is m, rest with its ends on two digital scales. A block whose mass is M rests on the beam, its center one-fourth away from the beam's left end. What do the scales read ?
- If the system is in equilibrium, the net force must be 0 N:

$$\sum F_y = F_l + F_r - Mg - mg = 0$$



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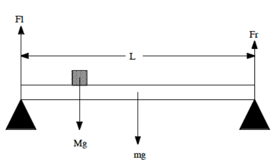
Equilibrium. Sample problem 1.

- If the system is in equilibrium, the net torque must be 0 Nm.
- Note: the toque associated with a force depends on the choice of the origin. The condition that the torque must be 0 Nm must be satisfied with respect to any choice of origin.
- If we choose the left scale as our origin, the left "scale" force does not appear in our torque equation:

$$\sum \tau_z = F_r L - Mg\frac{L}{4} - mg\frac{L}{2} = 0$$



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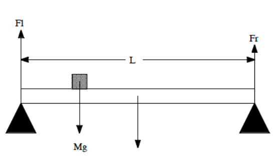
Equilibrium. Sample problem 1.

- The force generated by the right scale is thus equal to

$$F_r = \frac{Mg\frac{L}{4} + mg\frac{L}{2}}{L} = \frac{1}{4}Mg + \frac{1}{2}mg$$

- We can now use the first condition of equilibrium to determine the force generated by the left scale:

$$F_l = Mg + mg - F_r = \frac{3}{4}Mg + \frac{1}{2}mg$$



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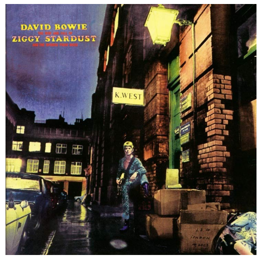
4 Minute 42 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 42 second intermission.

- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.



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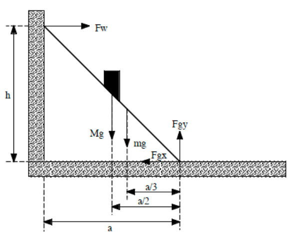
Equilibrium. Sample problem 2.

A ladder with length L and mass m rests against a wall. Its upper end is a distance h above the ground. The center of gravity of the ladder is one-third of the way up the ladder. A firefighter with mass M climbs halfway up the ladder. Assume that the wall, but not the ground, is frictionless. What is the force exerted on the ladder by the wall and by the ground?



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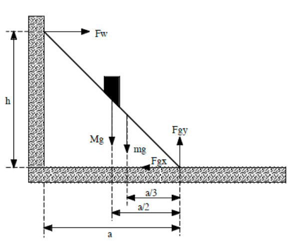
Equilibrium. Sample problem 2.

- Forces exerted by the wall and the floor:
  - The wall exerts a horizontal force (normal force).
  - The floor exerts a vertical force (normal force) and a horizontal force (friction force).
- Note: the friction force must be present in order to ensure that the net force in the horizontal direction add up to 0 N.



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Equilibrium. Sample Problem 2.

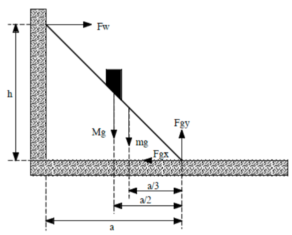
- The first condition for equilibrium requires that

$$\sum F_x = F_W - F_{gx} = 0$$

and

$$\sum F_y = F_{gy} - Mg - mg = 0$$

- Two equations with three unknown. We need more information! But we still have the second condition for equilibrium.



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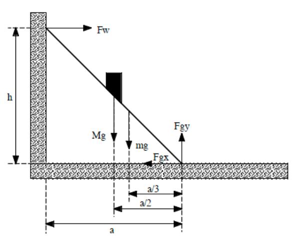
Equilibrium. Sample Problem 2.

- The second condition for equilibrium requires:

$$\sum \tau = hF_W - Mg\frac{a}{2} - mg\frac{a}{3} = 0$$

- Note: we have used the resting point on the ground as our reference point. The torque due to the two forces acting on this point do not contribute to the torque with this choice of reference point. We can now determine F_W easily:

$$F_W = \frac{ga}{h}\left(\frac{1}{2}M + \frac{1}{3}m\right)$$



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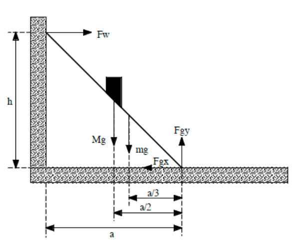
Equilibrium. Sample Problem 2.

- By examining the net force in the horizontal direction, we can determine the friction force:

$$F_{gx} = F_W = \frac{ga}{h}\left(\frac{1}{2}M + \frac{1}{3}m\right)$$

- Note: the frictional force depends on the position of the firefighter and increases when the fire fighter climbs the ladder.
- Since the frictional force must be less than $\mu_s F_{gy}$, there may be a maximum height that can be reaches by the fire fighter above which the ladder will slip.



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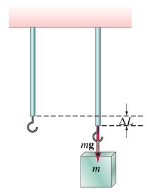
Stress and strain. The effect of applied forces.

- When we apply a force to an object that is kept fixed at one end, its dimensions can change.
- If the force is below a maximum value, the change in dimension is proportional to the applied force. This is called Hooke's law:

  $F = k\,\Delta L$

- This force region is called the **elastic region**.



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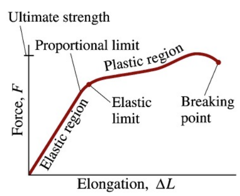
Stress and strain. The effect of applied forces.

- When the applied force increases beyond the elastic limit, the material enters the **plastic region**.
- The elongation of the material depends not only on the applied force F, but also on the type of material, its length, and its cross-sectional area.
- In the plastic region, the material does not return to its original shape (length) when the applied force is removed.



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Stress and strain. The effect of applied forces.

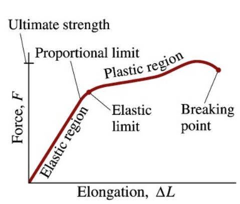
- The elongation ΔL can be specified as follows:

$$\Delta L = \frac{1}{E}\frac{F}{A} L_0$$

where

L_0 = original length
A = cross sectional area
E = Young's modulus

- **Stress** is defined as the force per unit area (= F/A).
- **Strain** is defined as the fractional change in length ($\Delta L_0/L_0$).



Note: the ratio of stress to strain is equal to the Young's Modulus.

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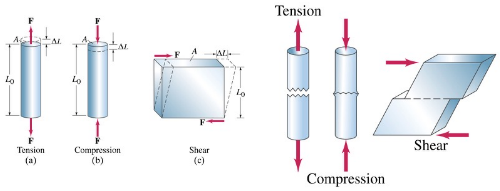
Stress and strain.
Direction matters.



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Stress and strain. A simple calculation could have prevented the death of 114 people.

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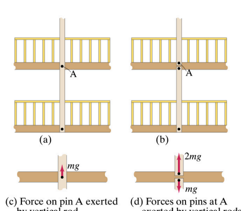
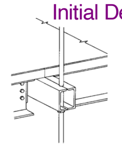
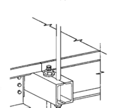
Stress and strain. A simple calculation could have prevented the death of 114 people.



Initial Design





As built

Credit: http://www.glendale-h.schools.nsw.edu.au/faculty_pages/ind_arts_web/bridgeweb/Hyatt_page.htm

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Done for today!
On Thursday: harmonic motion!

NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: ED99-45243-01 Date: 1999 Photo by: NASA

X-43A Hypersonic Experimental Vehicle - Artist Concept in Flight

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