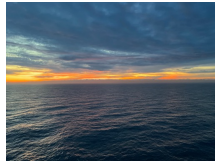
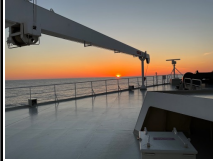


Physics 121. Lecture 16.



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Physics 121. Lecture 16.

- Course Information
- Quiz
- Topics to be discussed today:
 - Rotational Variables (Review)
 - Torque
 - Rolling Motion

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Physics 121. Lecture 16.

- Homework set # 6 is now available on the WEB and will be due on Saturday morning, April 4, at 8.30 am.
- There will be no workshops or office hours today and tomorrow.
- Exam 2 will be returned during workshops next week.

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Quiz.

- Practice what we have learned.



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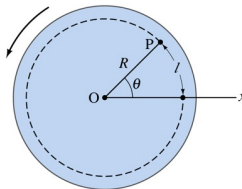
Rotational variables. A quick review.

- The variables that are used to describe rotational motion are:

- Angular position θ
- Angular velocity $\omega = d\theta/dt$
- Angular acceleration $\alpha = d\omega/dt$

- The rotational variables are related to the linear variables:

- Linear position $l = R\theta$
- Linear velocity $v = R\omega$
- Linear acceleration $a = R\alpha$



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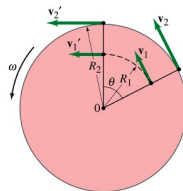
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Rotational variables. A quick review.

- Things to consider when looking at the rotation of rigid objects around a fixed axis:

- Each part of the rigid object has the same angular velocity.
- Only those parts that are located at the same distance from the rotation axis have the same linear velocity.
- The linear velocity of parts of the rigid object increases with increasing distance from the rotation axis.



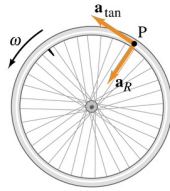
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Rotational variables. A quick review.

- Note: the acceleration $a_t = r\alpha$ is only one of the two components of the acceleration of point P. The two components of the acceleration of point P are:



- The radial component: this component is always present since point P carried out circular motion around the axis of rotation.
- The tangential component: this component is present only when the angular acceleration is not equal to 0 rad/s².

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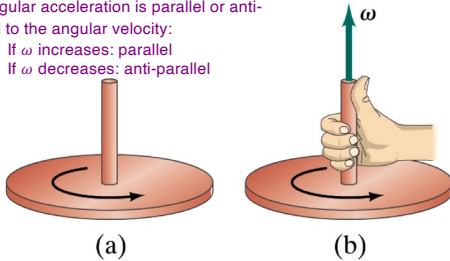
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Rotational variables. A quick review.

Angular velocity and acceleration are vectors! They have a magnitude and a direction. The direction of ω is found using the right-hand rule. The angular acceleration is parallel or anti-parallel to the angular velocity:

- If ω increases: parallel
- If ω decreases: anti-parallel



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Rotational kinetic energy.

- Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} (\sum_i m_i r_i^2) \omega^2$$

- The kinetic energy is proportional to the square of the rotational velocity ω . Note: the equation is similar to the translational kinetic energy ($\frac{1}{2} m v^2$) except that instead of being proportional to the mass m of the object, the rotational kinetic energy is proportional to the **moment of inertia I** of the object:

$$I = \sum_i m_i r_i^2 \Rightarrow K = \frac{1}{2} I \omega^2$$

Note: units of I : kg m²

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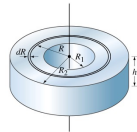
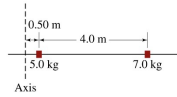
The moment of inertia. Calculating I .

- The moment of inertia of an object depends on the mass distribution of object and on the location of the rotation axis.
- For discrete mass distribution it can be calculated as follows:

$$I = \sum_i m_i r_i^2$$

- For continuous mass distributions we need to integrate over the mass distribution:

$$I = \int r^2 dm$$



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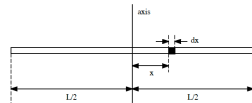
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Moment of inertia. Sample problem.

- Consider a rod of length L and mass m . What is the moment of inertia with respect to an axis through its center of mass?
- Consider a slice of the rod, with width dx , located a distance x from the rotation axis. The mass dm of this slice is equal to

$$dm = \frac{m}{L} dx$$



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Moment of inertia. Sample problem.

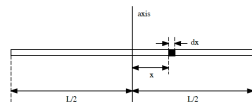
- The moment of inertia dI of this slice is equal to

$$dI = x^2 dm = \frac{m}{L} x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:

$$I = \int_{-L/2}^{L/2} dI = \int_{-L/2}^{L/2} \frac{m}{L} x^2 dx =$$

$$= \frac{m}{L} \left(\frac{1}{3} \left(\frac{L}{2} \right)^3 - \frac{1}{3} \left(-\frac{L}{2} \right)^3 \right) = \frac{1}{12} mL^2$$



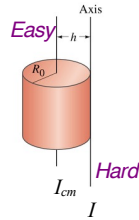
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Moment of inertia. Parallel-axis theorem.

- Calculating the moment of inertia with respect to a symmetry axis of the object is in general easy.
- It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.
- However, we can make a hard problem easier by using the parallel-axis theorem:



$$I = I_{cm} + Mh^2$$

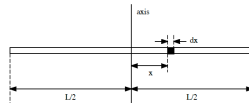
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Moment of inertia. Sample problem.

- Consider a rod of length L and mass m . What is the moment of inertia with respect to an axis through its left corner?
- We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:



$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$$

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Moment of inertia. Do not memorize them! You will get Figure 10.21 on our exams!

Object	Location of axis	Moment of inertia
(a) Thin hoop, radius R_0	Through center	MR_0^2
(b) Thin hoop, radius R_0 , width w	Through central diameter	$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c) Solid cylinder, radius R_0	Through center	$\frac{1}{2}MR_0^2$
(d) Hollow cylinder, inner radius R_1 , outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e) Uniform sphere, radius R_0	Through center	$\frac{2}{5}MR_0^2$
(f) Long uniform rod, length L	Through center	$\frac{1}{12}ML^2$
(g) Long uniform rod, length L	Through end	$\frac{1}{3}ML^2$
(h) Rectangular thin plate, length L , width w	Through center	$\frac{1}{12}M(L^2 + w^2)$

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4 Minute 9 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 9 second intermission.

- You can:
 - Stretch out.
 - Talk to your neighbors.
 - Ask me a quick question.
 - Enjoy the fantastic music.



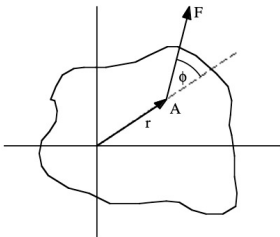
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Torque.

- Consider a force $\vec{F}$ applied to an object that can only rotate.
- The force $\vec{F}$ can be decomposed into two components:
 - A radial component directed along the direction of the position vector r . The magnitude of this component is $F\cos\phi$. This component will not produce any motion.
 - A tangential component, perpendicular to the direction of the position vector r . The magnitude of this component is $F\sin\phi$. This component will result in rotational motion.



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Torque.

- If a mass m is located at the position on which the force is acting (and we assume any other masses can be neglected), it will experience a linear acceleration equal to $F\sin\phi/m$.
- The corresponding angular acceleration is equal to $F\sin\phi/mr$.
- Since in rotational motion the moment of inertia plays an important role, we will rewrite the angular acceleration in terms of the moment of inertia:

$$\alpha = \frac{rF\sin\phi}{mr^2} = \frac{rF\sin\phi}{I}$$

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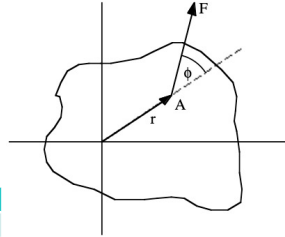
Torque.

- Consider rewriting the previous equation in the following way:

$$rF\sin\phi = I\alpha$$
- The left-hand-side of this equation is called the torque τ of the force F :

$$\tau = I\alpha$$
- This equation looks similar to Newton's second law for linear motion:

Linear	Rotational
Mass m	Moment I
Force F	Torque τ



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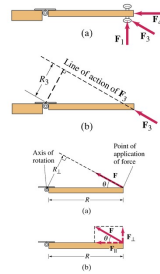
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Torque.

- In general, the torque associated with a force F is equal to

$$|\vec{\tau}| = rF\sin\phi = |\vec{r} \times \vec{F}|$$
- The arm of the force (also called the moment arm) is defined as $r\sin\phi$. The arm of the force is the perpendicular distance of the axis of rotation from the line of action of the force.
- If the arm of the force is 0, the torque is 0, and there will be no rotation.
- The maximum torque is achieved when the angle ϕ is 90° .



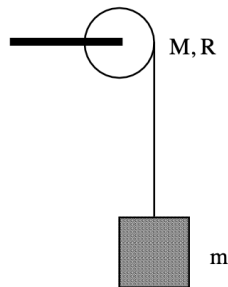
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Rotational motion. Sample problem.

- Consider a uniform disk with mass M and radius R . The disk is mounted on a fixed axle. A block with mass m hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension of the cord.



- Expectations:
 - Linear acceleration should approach g when M approaches 0 kg.

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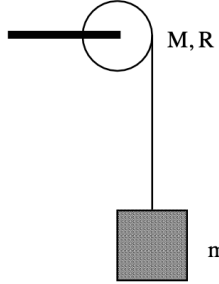
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Rotational motion. Sample problem.

- Start with considering the forces and torques involved.
- Define the sign convention to be used.
- The block will move down and we choose the positive y axis in the direction of the linear acceleration.
- The net force on mass m is equal to

$$ma = mg - T$$



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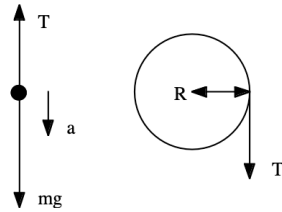
Rotational motion. Sample problem.

- The net torque on the pulley is equal to
- The resulting angular acceleration is equal to

$$\alpha = \frac{\tau}{I} = \frac{RT}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

- Assuming the cord is not slipping we can determine the linear acceleration:

$$a = \frac{2T}{M}$$



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Rotational motion. Sample problem.

- We now have two expressions for a :

$$a = \frac{2T}{M}$$

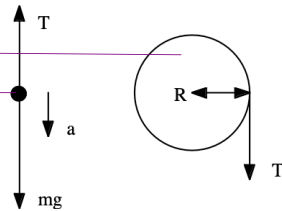
$$a = g - \frac{T}{m}$$

- Solving these equations we find:

$$T = \frac{M}{M + 2m}mg$$

$$a = \frac{2m}{M + 2m}g$$

Note: $a = g$ when $M = 0$ kg!!!



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Rolling motion.

- Rolling motion is a combination of translational and rotational motion.
- The kinetic energy of rolling motion has thus two contributions:
 - Translational kinetic energy:

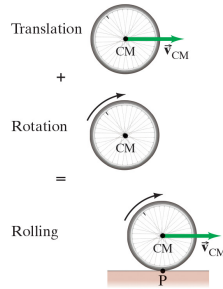
$$\frac{1}{2} M v_{cm}^2$$

- Rotational kinetic energy:

$$\frac{1}{2} I_{cm} \omega^2$$

- Assuming the wheel does not slip:

$$\omega = \frac{v}{R}$$



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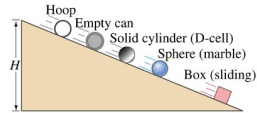
Rolling motion.

- Consider two objects of the same mass but different moments of inertia, released from rest from the top of an inclined plane:

- Both objects have the same initial mechanical energy (assuming their CM is located at the same height).

- At the bottom of the inclined plane, they will have both rotational and translational kinetic energy.

- Which object will reach the bottom first?



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Rolling motion.

- Initial mechanical energy:

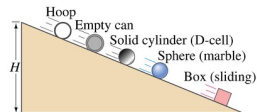
$$E_i = mgH$$

- Final mechanical energy:

$$E_f = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

- Assuming no slipping, we can rewrite the final mechanical energy as

$$E_f = \frac{1}{2} \left(m + \frac{I_{cm}}{R^2} \right) v_{cm}^2$$



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Rolling motion.

- Conservation of energy implies:

$$\frac{1}{2} \left(m + \frac{I_{cm}}{R^2} \right) v_{cm}^2 = m g H$$

or

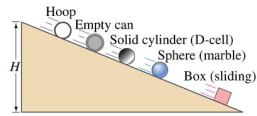
$$\frac{1}{2} \left(1 + \frac{I_{cm}}{m R^2} \right) v_{cm}^2 = g H$$

- The smaller I_{cm} , the larger v_{cm} at the bottom of the incline.

- Example:

- Ring: $I_{cm} = M R^2$, $v_{cm} = \sqrt{g H}$

- Disk: $I_{cm} = \frac{1}{2} M R^2$, $v_{cm} = \sqrt{\frac{4}{3} g H}$



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Done for today!

More about rotational motion next week.



N49's Cosmic Blast

Credit: Hubble Heritage Team ([STScI](#)/[AURA](#)), Y. Chu ([UJCU](#)) et al., NASA

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